PROBLEM SET # 3

Due February 19.

1. Let H be the group of upper triangular 3×3 matrices with 1-s on the main diagonal and $G = H \times S^1$. Let z be a non-identity element in the center $Z(H)$ and $t \in S^1$ be an element of infinite order. Consider the cyclic subgroup $\Gamma \subset G$ generated by (z, t).

(a) Prove that Γ is a normal closed subgroup of G.

(b) Prove that the commutator of the quotient G/Γ is not closed in G/Γ.

2. Let G be a compact Lie group with Lie algebra $\mathfrak{g}$. Assume that $\mathfrak{g}$ has trivial center. Prove that the center of G is finite.

3. Prove that $SU(n)$ is simply connected.

4. Show that $SU(2) \times SU(2)$ is isomorphic to the simply connected cover of $SO(4)$ and find the fundamental group of $SO(4)$. (Hint: recall the realization of $SU(2)$ by the set of quaternions of norm 1. Use the action of $SU(2) \times SU(2)$ on the space of quaternions by left and right multiplication and check that it induces surjection onto $SO(4)$.)

Date: February 10, 2020.