PROBLEM SET # 3

Let $\mathfrak{g}$ be a semisimple Lie algebra over $\mathbb{C}$

1. Show that the centralizer of any semisimple element in $\mathfrak{g}$ is reductive.

2. (Jacobson-Morozov Theorem) Let $x \in \mathfrak{g}$ be a nilpotent element. Show that there exist $h, y \in \mathfrak{g}$ such that $\{x, h, y\}$ form an sl_2-triple, i.e.

$$[h, x] = 2x, [h, y] = -2y, [x, y] = h.$$

Note that existence of h was proven in class.

3. Show that any two sl_2-triples containing x are conjugate by the action of the adjoint group.

4. Show that the nilpotent cone in $\mathfrak{g}$ has finitely many orbits with respect to the adjoint action.

Date: February 12, 2010.