POISSON LIE GROUPS AND QUANTUM GROUPS

1. Poisson Lie groups


Examples: dual space to a Lie algebra, co-adjoint orbits, moduli spaces of flat connections on a surface.

Poisson Lie group, tangent Lie bialgebra of a Poisson Lie group, exponential map for Lie bialgebras.

Example: standard Poisson Lie group structure on a Borel subgroup to a simple Lie group.

Quasitriangular Lie bialgebras, factorizable Lie bialgebras. The Drinfeld double of a Lie bialgebra. Corresponding Poisson Lie groups.


Actions of Poisson Lie groups. Symplectic leaves in Poisson Lie groups.

Examples: 1) standard Lie bialgebra structure on a simple Lie algebra, corresponding Poisson Lie groups, their symplectic leaves and the relation to double Bruhat cells.

If any time will be left after this:

• Kac-Moody Lie algebras. Standard Lie bialgebra structure. Lie bialgebras and Poisson Lie groups corresponding to loop algebras.
• Integrable Hamiltonian systems: Hamiltonian dynamics, integrable systems as Lagrangian fibrations. Construction of integrable systems from factorizable Poisson Lie groups.

2. Quantization


Theorem (EK): any Lie bialgebra can be deformed into a Hopf algebra.

2.2. \( U_q(g) \) and \( C_q(G) \). Quantization of the standard Lie bialgebra structure for simple Lie algebras.
3. Representation theory

3.1. Representation theory of $U_q(g)$ for generic $q$.
- Structure theory of $U_q(g)$, center and Harish-Chandra homomorphism, braid group action.
- Verma modules and highest weight theory, $R$-matrix.
- Kashiwara operators. Canonical bases and crystal bases. Global basis of $U_q(g)$.


References