POISSON LIE GROUPS AND QUANTUM GROUPS

1. Poisson Lie groups

1.1. **Poisson Lie groups and Lie bialgebras.** Elements of Poisson geometry: Poisson manifolds, symplectic manifolds, isotropic, Lagrangian and co-isotropic submanifolds. Symplectic leaves of a Poisson manifold. Presymplectic manifolds. Symplectic reduction of co-isotropic submanifolds. Symplectic reduction for a Hamiltonian group action. Marsden-Weinstein reduction.

Examples: dual space to a Lie algebra, co-adjoint orbits, moduli spaces of flat connections on a surface.

Poisson Lie group, tangent Lie bialgebra of a Poisson Lie group, exponential map for Lie bialgebras.

Example: standard Poisson Lie group structure on a Borel subgroup to a simple Lie group.

Quazitriangular Lie bialgebras, factorizable Lie bialgebras. The Drinfeld double of a Lie bialgebra. Corresponding Poisson Lie groups.

Examples: the standard Lie bialgebra structure on a simple Lie algebra. Belavin-Drinfeld Lie bialgebra structures on simple Lie algebra.

Actions of Poisson Lie groups. Symplectic leaves in Poisson Lie groups.

Examples: 1) standard Lie bialgebra structure on a simple Lie algebra, corresponding Poisson Lie groups, their symplectic leaves and the relation to double Bruhat cells.

If any time will be left after this:

- Kac-Moody Lie algebras. Standard Lie bialgebra structure. Lie bialgebras and Poisson Lie groups corresponding to loop algebras.
- Integrable Hamiltonian systems: Hamiltonian dynamics, integrable systems as Lagrangian fibrations. Construction of integrable systems from factorizable Poisson Lie groups.

2. Quantization

2.1. **Deformation quantization.** Deformations of algebras. Formal deformations. Deformation quantization of a Poisson algebra.

Bialgebras and Hopf algebras. Quasitriangular Hopf algebras. Braided monoidal categories. Deformations of bialgebras and of Hopf algebras. Deformation quantization of Hopf Poisson algebras.

Theorem (EK): any Lie bialgebra can be deformed into a Hopf algebra.

2.2. **$U_q(\mathfrak{g})$ and $C_q(G)$.** Quantization of the standard Lie bialgebra structure for simple Lie algebras.

3. Representation theory

3.1. Representation theory of $U_q(\mathfrak{g})$ for genetic q.

- Structure theory of $U_q(\mathfrak{g})$, center and Harish-Chandra homomorphism, braid group action.
- Verma modules and highest weight theory, R-matrix.
- Kashiwara operators. Canonical bases and crystal bases. Global basis of $U_q(\mathfrak{g})$.

3.2. Roots of unity. Different approaches: DeConcini–Kac–Procesi, Lusztig.

References

[1] Etingof, Schiffman, Lectures on quantum groups.
[2] Hong, Kang, Introduction to quantum groups and crystal bases.
[3] Theo Johnson Freyd lecture notes (unedited).
[4] Reshetikhin's notes on Poisson Lie groups and integrable systems.
[5] Jantzen, Lectures on quantum groups.
[6] Lusztig, Introduction to quantum groups.