

PROBLEM SET # 9
MATH 251

Due November 8.

1. Consider the group algebra $A = \mathbb{F}_3[S_3]$.

(a) Show that A has two simple modules, both have dimension 1.

(b) Show that A has two indecomposable projective modules and compute their dimensions.

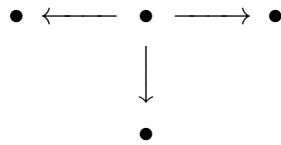
(c) Show that a projective resolution of a simple A -module is infinite, hence A has infinite global (homological) dimension. Hint: use Euler characteristic.

2.

(a) Let A be a k -algebra isomorphic to $B \times C$. Show that A has finite-representation type if and only if B and C have finite representation type.

(b) Let Q be a quiver. Show that Q is of finite type if and only if each connected component of Q is of finite type.

3. Classify indecomposable representations of the quiver



using Gabriel's theorem.

4. Show that the Kronecker quiver $\bullet \rightrightarrows \bullet$ has indecomposable representations in dimensions $(n, n + 1)$, (n, n) and $(n + 1, n)$. Then classify these representations up to isomorphism.