PROBLEM SET # 8
MATH 251

Due November 1.

1.(Lam 9.2) Can every finite group be realized as an irreducible linear group? In other words, is it true that any finite group has an irreducible faithful representation?

2.(Lam 9.4) Let k be a field of characteristic zero.

(a) Show that any unipotent subgroup $G \subset GL_n(k)$ is torsion free.

(b) If G is a maximal unipotent subgroup of $GL_n(K)$, show that G is a divisible group, i.e., for every $n > 0$ and $g \in G$ there exists $h \in G$ such that $h^n = g$.

3. (Lam 9.5) Let k be an algebraically closed and $G \subset GL_n(k)$ be a completely reducible linear group. Show that G is abelian iff G is conjugate to a subgroup of diagonal matrices in $GL_n(k)$.

4. Let $UT_3(\mathbb{C}) \subset GL_3(\mathbb{C})$ denote the subgroup of upper triangular matrices in $GL_3(\mathbb{C})$ with 1-s on the diagonal. Let G be the subset of $UT_3(\mathbb{C})$ consisting of matrices with integral entries.

(a) Show that G is a subgroup of $UT_3(\mathbb{C})$ and the center of G coincides with the commutator $[G, G]$. Moreover, the center is isomorphic to $\mathbb{Z}$.

(b) Show that G has a presentation with 3 generators a, b, c and relations

$$ab = cba, \quad ac = ca \quad bc = cb.$$

(c) Show for any $n > 0$, the group G has a complex irreducible representation of dimension n.

Date: October 25, 2016.