PROBLEM SET # 7
MATH 251

Due October 25.

1. Let G be a finite abelian group, and m be the least common multiple of orders of all elements. Show that k is a splitting field for G if and only if k contains a primitive m-th root of 1.

2. Let G be a finite abelian group, k be a splitting field for G.

(a) The set $\hat{G}$ of irreducible representations of G over k is a group with operation of tensor product.

(b) If the characteristic of k does not divide $|G|$, then $\hat{G}$ is isomorphic to G.

3. Let G and H be two finite groups, V and W be two absolutely simple $k[G]$ and $k[H]$-modules respectively.

(a) Prove that $V \otimes W$ is absolutely simple $k[G \times H]$-module. Note that the statement is true in any characteristic.

(b) Show by counterexample that if V and W are simple $k[G]$ and $k[H]$-modules respectively, the tensor product $V \otimes W$ may not be simple as $k[G \times H]$-module.

4. (Lam 8.21) Show that over $\mathbb{Q}$, $G = A_5$ has four irreducible representations of dimensions 1, 4, 5 and 6 respectively.

5. (Lam 8.23) If a finite group G has at most three irreducible complex representations, show that $G \simeq \{1\}, Z_2, Z_3$ or S_3.

6. (Lam 8.24) Suppose the character table of a finite group G has the following two rows:

g_1	g_2	g_3	g_4	g_5	g_6	g_7
1	1	1	ω^2	ω	ω^2	ω
2	-2	0	-1	-1	1	1

where $\omega = e^{2\pi i/3}$. Determine the rest of the character table.

Date: October 18, 2016.