PROBLEM SET # 6
MATH 251

Due October 18.

Below k is always a field.

1.(Lam 8.1) Give an example of a pair of non-isomorphic finite groups G and G' such that $k[G] \simeq k[G']$ for some field k.

2.(Lam 8.5) Let H be a normal subgroup of G. Show that $\operatorname{rad} k[H] = \operatorname{rad} k[G] \cap k[H]$.

3. (Lam 8.4) Let $R = k[G]$ for some field k and some group G. Let I be an ideal of R generated by $[R, R]$. Show that $R/I \simeq k[G/G']$ where G' is the commutator subgroup of G. Moreover, show that $I = \sum_{g \in G'} R(g - 1)$.

4. Let D_n be the dihedral group of order $2n$. Classify irreducible representations of D_n over $\mathbb{C}$, $\mathbb{R}$ and $\mathbb{Q}$.

5. Let k be a field of characteristic 2. Classify irreducible representations of the symmetric group S_4 over k.