PROBLEM SET # 5
MATH 251

Due October 11.

1. (Lam 6.5) Let k be uncountable infinite field. Show that for any group G, $\operatorname{rad} k[G]$ is a nil ideal of $k[G]$. (Hint: reduce the question to the case of a finitely generated group.)

2. (Lam 7.2) Let R be a finite-dimensional k-algebra which splits over k. Show that for any field $F \supset k$, $\operatorname{rad}(R^F) = (\operatorname{rad} R)^F$.

3. (Lam 7.5) Let R be a finite-dimensional k-algebra which splits over k. Show that any k-subalgebra of the center $Z(R)$ also splits over k.

4. (Lam 7.9) Let $F \supset k$ be a splitting field for a finite-dimensional k-algebra R, does it follow that F is also a splitting field for any quotient algebra of R.

5. Let B_n be the subalgebra of $M_n(k)$ consisting of all upper triangular matrices.

(a) Let $V = k^n$ be the natural representation of B_n obtained by restriction from $M_n(k)$. Show that any indecomposable B_n-module is isomorphic to some subquotient of B_n. (Hint: show that if M is an indecomposable B_n-module and $E_{n,n}M \neq 0$, then M is isomorphic to V. If $E_{n,n}M = 0$ proceed by induction on n.)

(b) Show that B_n has finite representation type.

6. Let V be a 3-dimensional vector space over $\mathbb{R}$ equipped with negative definite quadratic form q. Find all up to isomorphism simple modules of the Clifford algebra $\operatorname{Cliff}(V, q)$.

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