PROBLEM SET # 2
MATH 251

Due September 13.

1. Let R be a semisimple ring, $L \subset R$ be a left ideal. Prove that $L = Re$ for some $e \in R$ such that $e^2 = e$.

2. (Lam, 1.2.7) Show that for a semisimple module M over any ring the following conditions are equivalent

(1) M is finitely generated;
(2) M is noetherian;
(3) M is artinian;
(4) M is a finite direct sum of simple modules.

3. Let M be a semisimple left R-module, $B = \text{End}_R(M)$.

(a) If $S \subset M$ is a simple R-submodule, then for any non-zero $s \in S$ the right B-submodule sB is simple.

(b) M is a semisimple right B-module.

4. (Lam, 1.3.10) Show that a semisimple ring is Dedekind-finite.

5. Let R be a k-algebra for some algebraically closed field k and M be a simple R-module.

(a) If R is finite-dimensional k-algebra, then $\text{End}_R(M) = k$.

(b) Prove (a) in case when R is a countable-dimensional $\mathbb{C}$-algebra. (Hint: $\mathbb{C}(x)$ is not countable-dimensional over $\mathbb{C}$.)

Date: September 6, 2016.