PROBLEM SET # 11
MATH 251

Due November 22.

1. Show that any algebra over a field k can be embedded into a left primitive ring.

2. (Lam 11.4) Which of the following implications are true?

(a) R is left primitive if and only if $M_n(R)$ is left primitive.

(b) R is left primitive if and only if $R[t]$ is left primitive.

3. (Lam 11.7) Let V be a right module over a division ring k and $E = \mathrm{End}(V_k)$. Let R be a subring of E and I be a non-zero ideal in R. Show that R is dense in E if and only if I is dense in E,

4. (Lam 11.14) Let R be a left primitive ring such that $1 + r^2$ is a unit for any $r \in R$. Show that R is a division ring.

5. (Lam 12.1) Let R be a subdirectly irreducible ring. Show that if R is semiprimitive (resp. semiprime, reduced), then R is left primitive (resp. prime, a domain). In particular, R is left primitive if and only if R is right primitive.

Date: November 15, 2016.