PROBLEM SET # 10
MATH 251

Due November 15.

1. (Lam 10.1) For any semiprime ring R, show that the center $Z(R)$ is reduced and that characteristic of R is either zero or a square free integer.

2.(Lam 10.2) Let $\mathcal{P} \subset R$ be a prime ideal, I be a left ideal and J be a right ideal. Does $IJ \subset \mathcal{P}$ imply $I \subset P$ or $J \subset P$?

3. (Lam 10.4) Show that in a right artinian ring every prime ideal is maximal.

4. (Lam 10.4*) For any division ring k, list all prime and semiprime ideals in the subalgebra of upper triangilar matrices in $M_3(k)$.

5. (Lam 10.10) Let $N_1(R)$ be the sum of all nilpotent ideals of a ring R.

(a) Show that $N_1(R)$ is a nil subideal of Nil_*R.

(b) Show that if $N_1(R)$ is nilpotent, then $N_1(R) = Nil_*R$.

(c) Show that $N_1(R)$ is nilpotent, if all ideals in R satisfies DCC.

(d) Give an example of a ring R such that $N_1(R) \neq Nil_*R$.

Date: November 8, 2016.