FINAL EXAM
MATH 251

This exam is due on Thursday December 9 before midnight. You may email the exam or slide it under the door of my office. I am sorry but exams submitted after the deadline will not be graded. You do not have to solve all the problems to get A for this course,

1. Let F be a perfect algebraically closed field of characteristic $p > 0$ and $R = A_n(F)$ be the Weyl algebra with generators $x_1, \dots, x_n, y_1, \dots, y_n$ satisfying the relations

$$[x_i, x_j] = [y_i, y_j] = 0, \quad [x_i, y_j] = \delta_{ij}.$$

a) Show that the center of R is the polynomial algebra $F[x_1^p, \dots, x_n^p, y_1^p, \dots, y_n^p]$.

b) Show that any left primitive ideal of R has finite codimension.

c) Classify simple left R-modules and left primitive ideals of R.

d) Is it true that any prime ideal of R is primitive?

2. Let R be a ring and S be its subring.

a) Show that if M is a projective S-module, then $R \otimes_S M$ is a projective R-module.

b) Let $R = k[G]$ and $S = k[H]$ where G is a finite group, H is a subgroup and k is a field such that char k does not divide $|H|$. Then for any S-module M, the R-module $R \otimes_S M$ is projective.

c) Let G be a group (maybe infinite), P and M be $k[G]$-modules such that P is projective and M is finite-dimensional. Prove that the $k[G]$-module $P \otimes_k M$ is projective.

3. Let $G = A_5$ be the subgroup of even permutations of S_5 and k be a field of characteristic 5.

a) Classify irreducible representations of G over k. Is the prime field $\mathbb{F}_5$ a splitting field of G?

b) Classify indecomposable projective representations of G over k and describe the radical filtration of these representations. (The previous problem may be useful.)

c) Compute the Ext quiver for the category of $k[G]$-modules.

4. Show that there are infinitely many non-isomorphic centrally finite division rings with center $\mathbb{Q}$.

5. Give an example of a centrally finite division ring D non-isomorphic to D^{op}.

Date: December 1, 2016.