HOMEWORK ASSIGNMENT 6
MATH 250

October 24.

1. Let R be a semisimple ring, $L \subset R$ be a left ideal. Prove that $L = Re$ for some $e \in R$ such that $e^2 = e$.

2. Let P be a cyclic projective module over arbitrary ring R. Prove that $P = Re$ for some $e \in R$ such that $e^2 = e$.

3. Give an example of an infinite family $\{M_i\}_{i \in I}$ of simple modules such that $\prod_{i \in I} M_i$ is not semisimple.

4. Using description of finitely generated modules over principal rings, obtain a normal form of a linear operator in a finite-dimensional vector space over $\mathbb{R}$. Hint: irreducible (prime) elements in $\mathbb{R}[X]$ have degree 1 or 2.

5. (Lang, 3.23) Let $\{M_i\}$ be a directed family of modules over a ring R. For any module N show that

$$\lim_{\leftarrow} \mathrm{Hom}_R(N, M_i) = \mathrm{Hom}_R(N, \lim_{\leftarrow} M_i).$$

6. (Lang, 3.24) Any module is a direct limit of finitely generated submodules.

Date: October 17, 2014.