PROBLEM SET # 8
MATH 249

Due October 31.

1. Classify finite Eulerian semimodular lattices.

2. Let S be a finite set and $\rho\colon 2^S \to \mathbb{N}$ be a function satisfying

$$\rho(X) \le \rho(Y) \text{ if } X \subset Y,\ \rho(X) \le |X|,\ \rho(X \cap Y) + \rho(X \cup Y) \le \rho(X) + \rho(Y).$$

Call $X \subset S$ *closed* if for any Y containing X as a proper subset $\rho(X) < \rho(Y)$. Prove that the poset of closed sets with order defined by inclusion is a geometric lattice and $X \wedge Y = X \cap Y$.

3. Let A be a hyperplane arrangement without parallel cuts, i.e. for any cut $u \in L(\mathrm{A})$ of dimension greater than zero and any hyperplane $h \in \mathrm{A}$ the intersection of h and u is not empty. Let $h \in \mathrm{A}$ and V_h be the number of vertices (cuts of dimension zero) which do not belong to h. Show that the number of bounded regions equals

$$\sum_{v \in V_h} |\mu\left(\widehat{0}, v\right)|.$$

Date: October 25, 2006.