

**PROBLEM SET # 7**  
**MATH 249**

Due October 19.

1. Let  $L$  be a finite geometric distributive lattice. Prove that  $L$  is isomorphic to a Boolean lattice.

2. Prove that any interval of a geometric lattice is a geometric lattice.

3. Prove that in a geometric lattice  $\mu(x, y) \neq 0$  for any  $x \leq y$ .

4. Prove that the posets listed below with order given by inclusion are all lattices. Find among them all modular and geometric lattices.

- (1) Faces of a convex polytope;
- (2) Subgroups of a finite group;
- (3) Normal subgroups of a finite group.