PROBLEM SET # 6
MATH 249

Due October 12.

1. Denote by $P+Q$ the disjoint union of two posets P and Q, by P^* the poset on P with reversed order and by $P \times Q$ the direct product of P and Q. If A and B are algebras over a field k, then $A \oplus B$ is their direct sum, $A \otimes B$ is their tensor product and A^{op} is the algebra on the same space as A with reversed multiplication. Finally, let $I(P)$ denote the incidence algebra of a poset P. Prove that

$$I(P+Q) \cong I(P) \oplus I(Q),\ I(P^*) \cong I(P)^{\mathrm{op}}$$

for locally finite P and Q, and

$$I(P \times Q) = I(P) \otimes I(Q)$$

for finite P and Q.

2. Let $L_n(\mathbb{F}_q)$ be the set of all subspaces in $\mathbb{F}_q^n$ ordered by inclusion. Find the Möbius function for this poset. Hint: Problem 2 homework 3.

3. Prove that any Cohen-Macauley poset is graded.

4. Give an example of an Eulerian poset which is not Cohen-Macauley.

Date: October 5, 2006.