Due October 12.

1. Denote by $P + Q$ the disjoint union of two posets $P$ and $Q$, by $P^*$ the poset on $P$ with reversed order and by $P \times Q$ the direct product of $P$ and $Q$. If $A$ and $B$ are algebras over a field $k$, then $A \oplus B$ is their direct sum, $A \otimes B$ is their tensor product and $A^{op}$ is the algebra on the same space as $A$ with reversed multiplication. Finally, let $I(P)$ denote the incidence algebra of a poset $P$. Prove that

$$I(P + Q) \cong I(P) \oplus I(Q), \quad I(P^*) \cong I(P)^{op}$$

for locally finite $P$ and $Q$, and

$$I(P \times Q) = I(P) \otimes I(Q)$$

for finite $P$ and $Q$.

2. Let $L_n(F_q)$ be the set of all subspaces in $F_q^n$ ordered by inclusion. Find the Möbius function for this poset. Hint: Problem 2 homework 3.

3. Prove that any Cohen-Macaulay poset is graded.

4. Give an example of an Eulerian poset which is not Cohen-Macaulay.

Date: October 5, 2006.