PROBLEM SET # 5
MATH 249

Due October 5.

1. Let L be a symmetric matrix such that l_{ij} are non-positive integers for all $i \neq j$ and the sum of entries in each row is zero. Is it true that L is the Laplace matrix of some graph?

2. Let L be the Laplace matrix of a graph Γ. Prove that all eigenvalues of L are non-negative and not greater than the double degree of some vertex in Γ.

3. Let $e_1, \ldots, e_n$ be a basis in $\mathbb{R}^n$ and P be the convex hull of $\pm e_1, \ldots, \pm e_n$. Let Γ be the graph of the convex polytope P. Find the complexity of Γ.

4. Let $f(n)$ be the number of ways to glue a torus from the $2n$-gon. Find $f(n)$.

Date: September 28, 2006.