PROBLEM SET # 4
MATH 249

Due September 28.

1. Show that the number of plane binary trees with $n + 1$ end vertices equals the Catalan number $c(n)$.

2. Find the number of correct expression with n pairs of brackets and k pairs of parentheses. Write the generating function.

3. Prove the following identity

$$\prod_{i=1}^{s} \left(1 + x^{-1}q^i\right) \prod_{j=0}^{t-1} \left(1 + xq^j\right) = \sum_{k=-s}^{t} \binom{s+t}{s+k}_q q^{k(k-1)/2} x^k.$$

4. Prove the Jacobi triple product identity using the identity in Problem 3.