

PROBLEM SET #3
MATH 249

Due September 21

1. Using Euler identity prove the following recurrent relation for the number of partition $p(n)$

$$p(n) = \sum_{i \geq 1} (-1)^{i-1} \left(p\left(n - \frac{(3i+1)i}{2}\right) + p\left(n - \frac{(3i-1)i}{2}\right) \right).$$

2. Prove the q-binomial identity

$$\prod_{i=0}^{n-1} (1 + xq^i) = \sum_{k=0}^n \binom{n}{k}_q x^k q^{k(k-1)/2}.$$

3. Prove the Gauss identity

$$\prod_{i \geq 1} \frac{1 - x^{2i}}{1 - x^{2i-1}} = \sum_{k \geq 1} x^{k(k-1)/2}$$

using the triple product Jacobi identity.

4 (Extra credit). Give a combinatorial proof of the above Gauss identity.