PROBLEM SET # 2
MATH 249

Due September 14.

1. Let $p_k(n)$ denote the number of partitions of n into k parts, $p(n)$ denote the number of all partitions of n.

(a) Fix k. Prove that there are two polynomials $F(x)$ and $G(x)$ of degree $k-1$ such that $F(n) < p_k(n) < G(n)$ for any integer positive n.

(b) Prove that $p(n)$ grows faster than any polynomial, i.e. for any polynomial $H(x)$ there is N such that $p(n) > H(n)$ for all $n > N$.

2. Let $l(s)$ denote the number of inversions in a permutation s.

(a) Prove that $l(s\tau) - l(s) = \pm 1$ for a permutation s and a transposition $\tau = (i, i+1)$.

(b) Prove that $l(st) \leq l(s) + l(t)$ for any two permutations s and t.

3. Let $A(n,k)$ be Euler numbers. Prove the identity

$$x^n = \sum_{k=1}^{n} A(n,k) \binom{x+k-1}{n}.$$

You will get extra credit for a combinatorial proof.

Date: September 6, 2006.