PROBLEM SET # 10
MATH 249

Due November 14.

1. Let

$$P = \left\{ (x_1, \dots, x_N) \in \mathbb{R}^N \mid 0 \le x_i \le 1, \sum_{i=1}^{N} x_i = 1 \right\}.$$

Show that P is a convex polytope and find $i(P, n)$ and $i^+(P, n)$.

2. Let P be a convex integral polytope in $\mathbb{R}^N$ and $f : \mathbb{R}^N \to \mathbb{R}^N$ be defined by the formula

$$f(x) = Ax + \beta,$$

where $\beta \in \mathbb{Z}^N$, A be a matrix with integral coefficients such that $\det A = \pm 1$. Prove that $i(P, n) = i(f(P), n)$.

3. Let $\operatorname{Par}(n)$ denote the set of partitions of n. Define a partial order on $\operatorname{Par}(n)$ by $\lambda \le \mu$ if $\lambda_1 + \cdots + \lambda_k \le \mu_1 + \cdots + \mu_k$ for all $k \ge 1$. Prove that $\operatorname{Par}(n)$ is a lattice.

4. Let $\lambda = (\lambda_1, \dots, \lambda_n) \in \operatorname{Par}(n)$ and P_λ be the convex hull of $\left(\lambda_{s(1)}, \dots, \lambda_{s(n)}\right)$ for all $s \in S_n$. Prove that $\lambda \le \mu$ iff $P_\lambda \subseteq P_\mu$.

Date: November 6, 2006.