Sample final

This sample final is longer than the real final.

1. Find the area of the surface obtained by rotating the curve

$$y = \sqrt{1 + e^x}, \ 0 \le x \le 1$$

about the *x*-axis.

2. Determine if the integral is convergent. If it is convergent evaluate the integral

$$\int_0^\infty \frac{dx}{e^x + e^{-x}}$$

3. Determine if the sequence is convergent. If it is convergent find the limit

$$\left\{n^{\frac{1}{n}}\right\}.$$

4. Determine whether the series is absolutely convergent, conditionally convergent or divergent

$$\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$$

5. Find the radius of convergence and the interval of convergence of the series

$$\sum_{n=2}^{\infty} \frac{x^n}{\ln n}.$$

6. Find the Taylor polynomial $T_5(x)$ of the fifth degree for $f(x) = \arcsin x$ at x = 0. Hint: use $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$. 7. Solve the initial value problem

$$y' = \frac{x}{y+1}, \ y(0) = 0.$$

8. Solve the equation

$$y' + y + e^{-x} = 0.$$

9. Solve the initial value problem

$$y'' - 2y' + 5y = 0, \ y(0) = 0, \ y'(0) = 3$$

10. Solve the differential equation

$$y'' - y = x^2 + e^x.$$

11. Solve the initial value problem in power series

$$y'' - 2xy' = 2y, y(0) = 0, y'(0) = 1.$$

12. Write the following complex numbers in the form a + bi

(a) $e^{\pi i/4}$; (b) $(1-i)^{10}$.

13. A spring with mass 2kg is stretched 0.5m beyond its original length and then released with zero velocity. The force of 5N is required to keep the spring stretched 0.5m beyond its original length. The damping constant is 8. Find the position of the mass as a function of time.

14. Solve the differential equation

$$y'' + 3y' + 2y = \cos(e^x).$$