1. Let
\[ f(x, y, z) = x^3 - y \tan z. \]
Find \( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}. \)

2. Evaluate the integral
\[ \int \int_R (x + y) \, dx \, dy \]
for \( R \) bounded by the line \( y = x \) and the parabola \( y = x^2 \).

3. Use partial derivatives to obtain the line equation which is the best to fit the data points \((-1, -1), (0, 1), (1, 2)\).

4. Solve the initial value problem
\[ y' = y^2 \sin x, \quad y(0) = 8. \]

5. A bank account has $50,000 earning 5% interest compounded continuously. An owner of the account withdraws money continuously at the annual rate of $4,000. How long will it take for the balance to drop to zero?

6. Approximate
\[ \int_0^1 \cos (x^2) \, dx. \]

7. Check for convergence
\[ (a) \sum_{k=1}^{\infty} \frac{1}{k^3 + 3k + 5}, \]
\[ (b) \sum_{k=1}^{\infty} \frac{1}{\sqrt{k^2 + 1}}. \]

8. Find \( f^{(7)}(0) \) for
\[ f(x) = \frac{\cos(x^2) - 1}{x}. \]

9. Suppose that the amount of time required to serve a customer in a bank has an exponential density function with mean 10 minutes. Find the probability that a customer will require more than 10 minutes.

10. A student can take a certain exam three times. If he fails all three times, he should leave the school. The probability that he passes exam is 80%. Find the probability that he will stay at this school.

\[ \text{Date: May 4, 2005.} \]
Solutions.

1. \[ \frac{\partial f}{\partial x} = 3x^2, \quad \frac{\partial f}{\partial y} = -\tan z, \quad \frac{\partial f}{\partial z} = \frac{y}{\cos^2 z}. \]

2. Rewrite the integral as \[ \int_0^1 \int_{x^2}^x (x + y) dy \, dx. \]

Then \[ \int_{x^2}^x (x + y) dy = xy + \frac{y^2}{2} \Big|_{x^2}^x = x^2 - x^3 + \frac{x^2 - x^4}{2} = \frac{3x^2}{2} - x^3 - \frac{x^4}{2} \]

\[ \int_0^1 \left( \frac{3x^2}{2} - x^3 - \frac{x^4}{2} \right) dx = \left[ \frac{x^3}{2} - \frac{x^4}{4} - \frac{x^5}{10} \right]_0^1 = \frac{3}{20} \]

3. Write the equation \( y = Ax + B. \) Then we have to minimize \( E = (B - A + 1)^2 + (B - 1)^2 + (B + A - 2)^2 = 3B^2 + 2A^2 - 4B - 6A + 6. \)

Write the equation for partial derivatives

\[ \frac{\partial E}{\partial A} = 4A - 6 = 0 \]

\[ \frac{\partial E}{\partial B} = 6B - 4 = 0 \]

Solve and obtain \( A = \frac{3}{2}, \quad B = \frac{2}{3}. \) The equation is \( y = \frac{3}{2}x + \frac{2}{3}. \)

4. First find a general solution

\[ \int \frac{dy}{y^2} = \int \sin x \, dx, \quad y = 0 \]

\[ -\frac{1}{y} = -\cos x + C \]

\[ y = \frac{1}{\cos x - C} \]

To find \( C \) put \( x = 0, \quad y = 8 \) and get the equation \( 8 = \frac{1}{1 - C} \)

Solve \( C = \frac{7}{8}. \)

The answer \( y = \frac{1}{\cos x - 7/8}. \)
5. Differential equation on the account balance \( y(t) \) is
\[
y' = 0.05y - 4,000, \ y(0) = 50,000
\]
First solve the equation. Integration factor is \( e^{-0.05t} \).
\[
\left( e^{-0.05t}y \right)' = -4,000e^{-0.05t}
\]
\[
e^{-0.05t}y = 80,000e^{-0.05t} + C
\]
\[
y = 80,000 + Ce^{0.05t}
\]
Use initial data \( y(0) = 50,000 \) to find \( C = -30,000 \). Therefore the account balance after \( t \) years is given by the formula
\[
y = 80,000 - 30,000e^{0.05t}.
\]
To answer the problem question solve for \( t \) the equation
\[
y = 80,000 - 30,000e^{0.05t}
\]
\[
e^{0.05t} = \frac{8}{3}
\]
\[
0.05t = \ln \frac{8}{3}
\]
\[
t = 20 \ln \frac{8}{3} \approx 19.6.
\]
6. Write the Taylor series for \( \cos(x^2) \) at \( x = 0 \)
\[
\cos(x^2) = 1 - \frac{x^4}{2} + \frac{x^8}{24} - \ldots
\]
Then write the series for an antiderivative
\[
\int \cos(x^2) \, dx = x - \frac{x^5}{10} + \frac{x^9}{216} - \ldots
\]
\[
\int_0^1 \cos(x^2) \, dx \approx 1 - \frac{1}{10} + \frac{1}{216} \approx 0.905
\]
7. In both cases use the comparison test. In the first case
\[
\frac{1}{k^3 + 3k + 5} \leq \frac{1}{k^3}.
\]
The series
\[
\sum_{k=1}^{\infty} \frac{1}{k^3}
\]
is convergent. Hence
\[
\sum_{k=1}^{\infty} \frac{1}{k^3 + 3k + 5}
\]
is convergent.
In the second case note that
\[ \sqrt{k^2 + 1} \leq \sqrt{k^2 + 2k + 1} = k + 1. \]
Therefore
\[ \frac{1}{\sqrt{k^2 + 1}} \geq \frac{1}{k + 1}. \]
Since
\[ \sum_{k=1}^{\infty} \frac{1}{k + 1} \]
is divergent then
\[ \sum_{k=1}^{\infty} \frac{1}{\sqrt{k^2 + 1}} \]
is divergent.

8. Use the Taylor series
\[ f(x) = -\frac{x^3}{2} + \frac{x^7}{24} - \ldots \]
If \( a_7 \) is the coefficient then \( a_7 = \frac{f^{(7)}(0)}{7!} \). In our case \( a_7 = \frac{1}{24} \). Hence
\[ f^{(7)}(0) = \frac{7!}{24} = 210. \]

9. The density function is \( f(x) = \frac{1}{10}e^{-\frac{x}{10}} \).
\[ \Pr(X > 10) = 1 - \Pr(X \leq 10) \]
\[ \Pr(X \leq 10) = \int_{0}^{10} \frac{1}{10}e^{-\frac{x}{10}}dx = -e^{-\frac{x}{10}}\bigg|_{0}^{10} = 1 - e^{-1}. \]
\[ \Pr(X > 10) = e^{-1} \approx 0.37. \]

10. The probability that he fails once is 0.2. The probability that he fails three times is \( (0.2)^3 = 0.008 \). Hence he will stay with probability \( 1 - 0.008 = 0.992 = 99.2\% \).