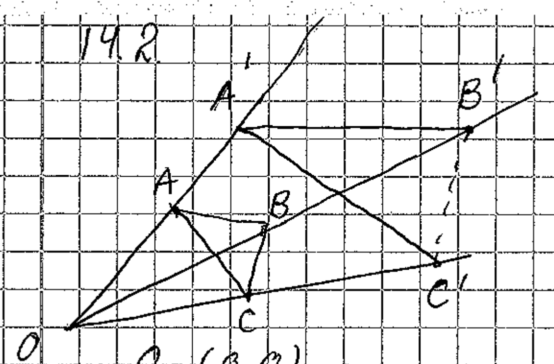
14.2



$O = (0,0)$

$A = (0,1) \Rightarrow A' = (0,a)$ for some $a \in F$

$B = (1,0) \qquad B' = (a,0)$ because $AB \parallel A'B'$

If $C = (x,y)$ then $C' = (ax, ay)$

because $AC \parallel A'C'$

The slope of $BC = \frac{y-1}{x}$

the slope of $B'C' = \frac{ay-a}{a} = \frac{y-1}{x} \Rightarrow BC \parallel B'C'$.

14.3. The slope of a line $ax+by+c=0$ is defined as $m = -b^{-1}a$

Check I1: $A = (a_1, a_2)$, $B = (b_1, b_2)$

~~ax+by~~ $\quad a a_1 + b a_2 + c = 0$

$a b_1 + b b_2 + c = 0$

$\Downarrow$

$a(b_1 - a_1) + b(b_2 - a_2) = 0$

So if $b_1 - a_1 \neq 0$ ~~put b = 1~~

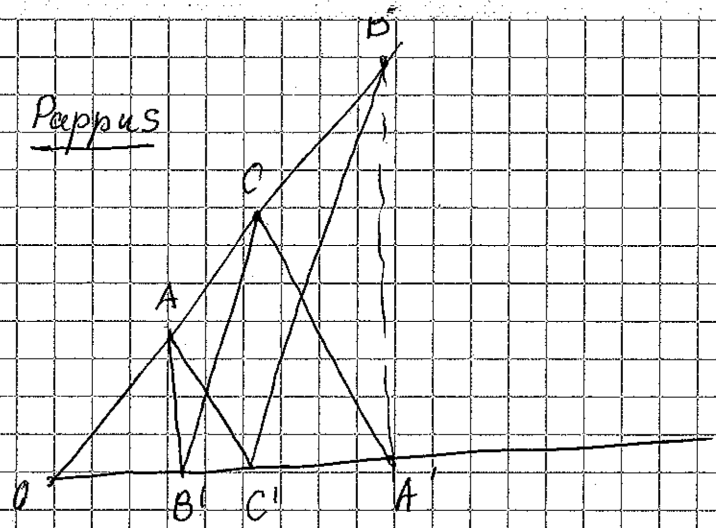
$a = -b(b_2 - a_2)(b_1 - a_1)^{-1}$ can be found uniquely as $c = -a a_1 - b a_2$

Check I2: $y = mx + c$ $\quad (0,c), (1, m+c)$ are on the line

Check I3: $(0,0), (0,1)\ (1,0)$ are non-collinear.

Two lines are parallel if they have the same slope or $a' = ta$, $b' = tb$ for some $t \in F$

Pappus



Let $O=(0,0)$, $A=(1,0)$, $B'=(0,1)$,

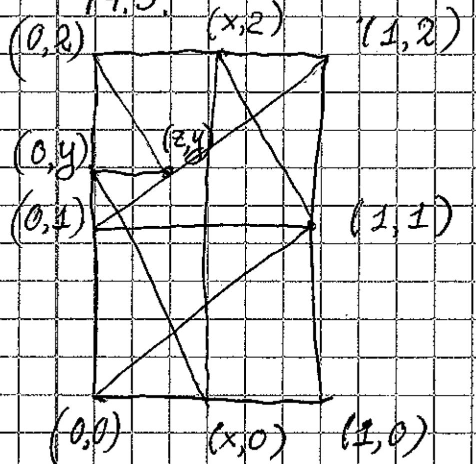
$C=(a,0)$, $C'=(0,b)$, $A'=(0,y)$, $B=(x,0)$

$AC' \parallel CA' \Rightarrow b = y a^{-1} \Rightarrow y = ba$

$BC' \parallel CB' \Rightarrow a^{-1} = b x^{-1} \Rightarrow a^{-1}x = b \Rightarrow x = ab$

$AB' \parallel A'B$ if and only if $x = y$. Hence $ab = ba$

14.5.



$$y = z + 1$$

$$\frac{-y}{x} = \frac{y-2}{z} = \frac{1}{x-1}$$

$$\frac{-y}{x} = \frac{y-2}{y-1} = \frac{1}{x-1}$$

$$y = \frac{x}{1-x}$$

$$\frac{1-x}{2x-1} + \frac{1}{x-1} = 1$$

$$\frac{y-2}{y-1} = 1 - \frac{1}{y-1}$$

$$\cancel{2x^2 - x = (x-1)^2 + x}$$

$$\cancel{x^2 - x = 3x - x^2 - 1}$$

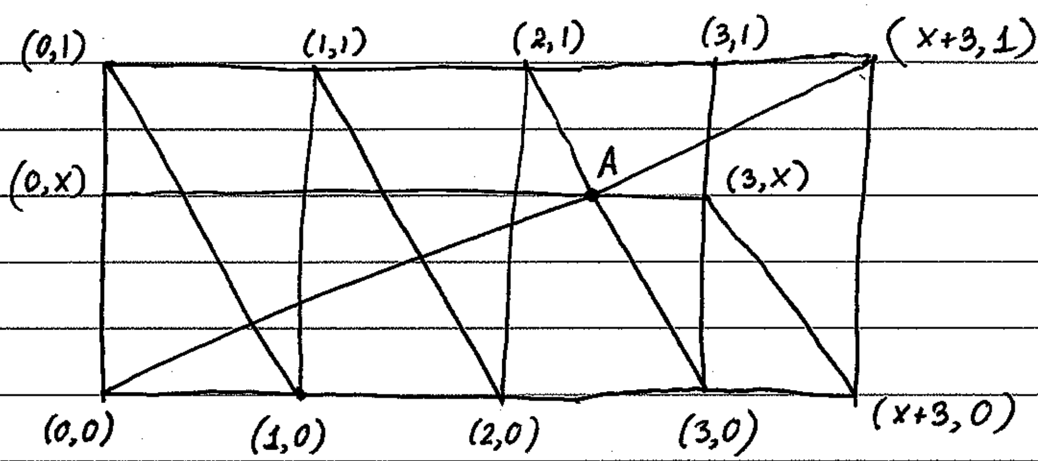
$$\cancel{2x^2 - 4x + 1 = 0}$$

$$\frac{1}{y-1} + \frac{1}{x-1} = 1$$

$3x^2 - 7x + 3 = 0$ $\quad D = 49 - 36 = 13$

$\sqrt{13} \in F$

14.6.



Coordinates of $A = (a, x)$

$a + x = 3$, $\quad \frac{a}{x} = x + 3 \Rightarrow a = x^2 + 3x$

$x^2 + 4x - 3 = 0 \qquad D = 4^2 + 4 \cdot 3 = 4 \cdot 7$

$\sqrt{7} \in F$.

15.1 Suppose $a^{-1} < 0$, then $-a^{-1} > 0$
$a \cdot (-a^{-1}) > 0 \Rightarrow -1 > 0 \Rightarrow 1 < 0$ contradiction.

15.2 If $b^2 = a$ then $(-b)^2 = 0$.
Since $x^2 - a$ has two roots, they are $\{b, -b\}$.

15.4. Let $S = \{\alpha \in F \mid \exists n \in \mathbb{Z} \text{ with } \alpha < n\}$ and $T = F - S$. Let c be such that $\alpha \leq c \leq \beta$ for all $\alpha \in S, \beta \in T$. If $c \in S$ then $c + 1 \in S$ $c + 1 \geq c$. Impossible. If $c \in T$ then $c - 1 \in T$ $c - 1 \leq c$. Also impossible. Hence $T = \emptyset$.