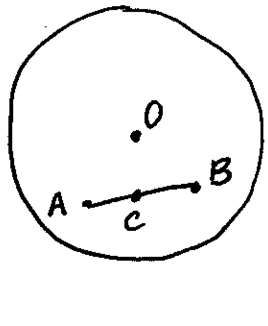
11.1



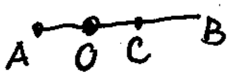
(a) Let R be the radius of Γ

Let $A * C * B$. Assume first that ~~them~~ the center of Γ does not lie on AB. Then either $\angle OCB$ or $\angle OCA$ is not less than RA.

Then if say $OCA \geq RA$, $\angle OAC < RA$ and by I. 19, $OC < OA < R$. So C is inside Γ.

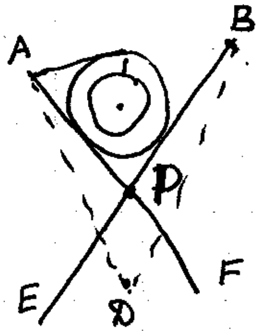
If O lies on AB.

then either $O*C*B$ or $O*C*A$. So $OC < OB$ or $OC < OA$.



C is inside Γ.

(b)



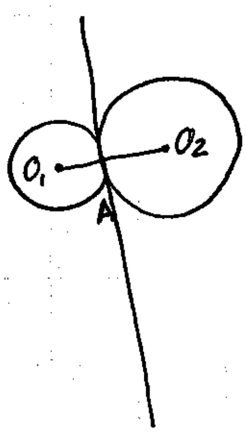
Through A and B construct tangent lines to Γ. Since there are two tangent lines through A at least one of them meets the tangent line through B at P

Let $\angle PEF$ be vertical to $\angle APB$ and D be insid $\angle EPF$

Since interior of Γ is inside $\angle APB$ and none of points ~~of~~ AD and BD is insid $\angle APB$,

both AD and BD are outside Γ.

11.2.



Two circles Γ_1 and Γ_2 with centers O_1 and O_2 are tangent at A if and only if O_1, A and O_2 are collinear. This happens iff the lines perpendicular O_1A and O_2A coinside, which is equivalent to the condition that the line tangent to Γ_1 ~~atd~~ A and the line tangent to Γ_2 at A coincide.

11.3.

Consider the centers G, D of circles Γ and Δ respectively. Then D, G and A are collinear and we have two cases without loss of generality

1) $D * G * A$

2) $D * A * G$ and D, X, A are not collinear.

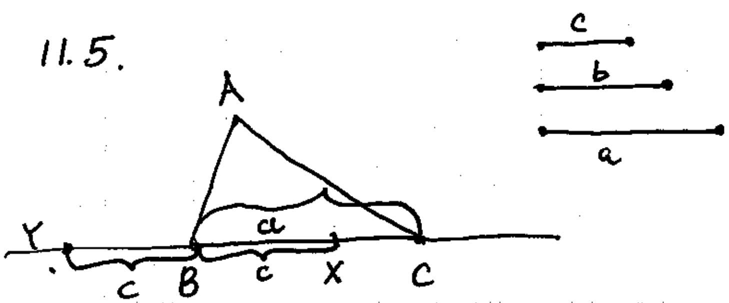
In the first case let $X \in \Gamma$. Then $GX \cong GA$ and $DX < DG + GX = DG + GA = DA$. So X is inside Δ.

(Remark: if $D\,X\,A$ are collinear, then $XD < XG = XA$)

In the second case let $X \in \Gamma$ and D, X, A are not collinear. Then $DX + XG > DG = DA + AG$. But $DX \cong DA$, So $XG > AC$ and X is outside Δ.

(Remark: collinear case is trivial in this situation)

11.5.



Without loss of generality assume that $a \geq b$, $a \geq c$

Let $BC \cong a$.

Construct the circle Γ centered at B and with radius c and the circle Δ centered at C and with radius b.

Let X be the point on $\overrightarrow{BC}$ such that $BX \cong c$.

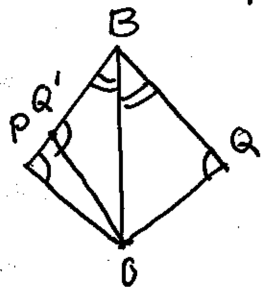
Then since $c \leq BC$ we have $BX \leq BC$ so either $X = C$ or $B * X * C$. In any case $XC \leq b$ since $b + c < a$. Thus, $X \in \Gamma$ and X lies inside Δ

Now let Y be on the opposide side of B and $BY \cong c$

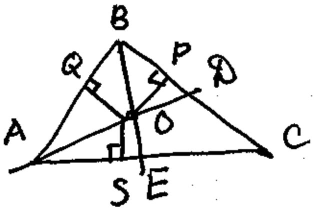
Then $YB + BC = a + c > b$. So $Y \in \Gamma$ and Y lies outside Δ.

Therefore Γ and Δ meet at a point A. So $BA \cong c$, $AC \cong b$ and $BC \cong a$.

11.6. <u>Proposition</u>. P and Q on opposite sides of BO. $\angle PBO \cong \angle QBO$. Then $\triangle BPO \cong \triangle BQO$.

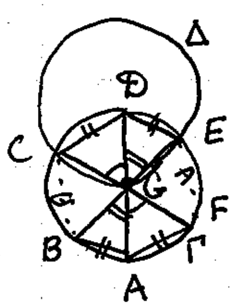


<u>Proof.</u> Assume $BP > BQ$. Then there is Q' on $\overrightarrow{BP}$ such that $BQ' \cong BQ$ and $P * Q' * B$. Then $\triangle BQ'O \cong \triangle BQO$ (C6). So $\angle BQ'O \cong \angle BQO \cong \angle BPO$. Contradiction since $\angle BQ'O > BPO$ (exterior angle).



Let BE be the angle bisector of $\angle ABC$ and $A\mathcal{D}$ be the angle bisector of $\angle BAC$.
By Crossbar $\overrightarrow{BE}$ meets $\overline{A\mathcal{D}}$ at O.
Let OP, OQ and OS be perpendicular to BC, AB and AC respectively. Then by the previous proposition $OP \cong OQ \cong OS$.
So the circle Γ with center O and radius OP is tangent to all 3 sides of $\triangle ABC$.

11.7.



Since A is outside Δ and $\mathcal{D}$ is inside Δ we have that Δ and Γ meet at two points C and E. By construction $C\mathcal{D} \cong \mathcal{D}E \cong CG \cong GE$.
$BG \cong GE$, $GF \cong GG$, so $\triangle ABG \cong \triangle \mathcal{D}EG$ (C6), similarly $\triangle AFG \cong \triangle \mathcal{D}CG$.
Thus we obtain $C\mathcal{D} \cong \mathcal{D}E \cong AB \cong AF$.

By (C6) we also have $\triangle GCB \cong CFE$, so $BC \cong EF$.

But to prove $BC \cong C\mathcal{D}$ we need (P).