6.1

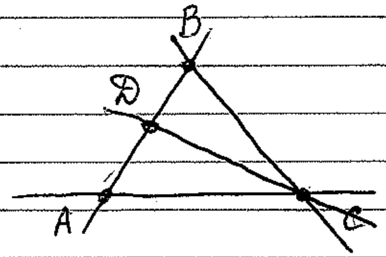
There are three points not on the same lin

Hence we have 3 lines

AB, BC and AC.

Now we have two cases:

1)

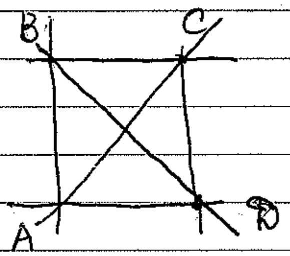


1) Let the forth point D lie ~~on~~ of AB, AC or BC.

without loss of generality, say D lies on AB

then we have one more line CD (no other lines since for each pair of points we already have a line

2)



2) D does not lie on AB, BC or AC.

Then we must have 3 more lines DB, DC, DA

This is F_2^2 geometry which we discussed in class

Both satisfy P!

6.4.

P1. Let A, B be two "points". They are two one-dimensional subspaces in F^3 generated by vectors v and w respectively. Vectors v, w are not proportional, they span the two-dimensional subspace W, which is a "line" in our geometry containing A and B.

P2. Given 2 two-dimensional subspaces W and W',

We have $\dim W \cap W' + \dim W + W' = 4$. Since

$\dim W + W' = 3$ $(W \neq W')$, $\dim W \cap W' = 1$. So 1-dimensional space

$W \cap W'$ is the "point" where "lines" W and W' meet.

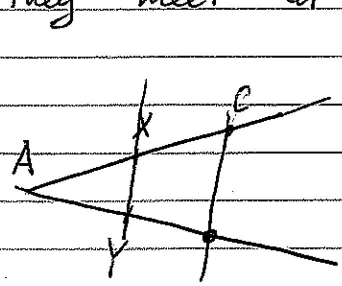
P3. Let W be a two-dimensional subspace, and $\{v, w\}$ form a basis in W. Then

Fv, Fw and $F(v+w)$ are 3 one-dimensional subspaces (3 "points") contained in W.

P4. Let v_1, v_2, v_3 be a basis in $V = F^3$. Then

Fv_1, Fv_2 and Fv_3 are three non-collinear "points".

6.5. (a) Let l and m be two lines. First assume that they meet at A. ~~For any $X \in l, Y \in m$ join XY~~ Pick up $X \in l, Y \in m$



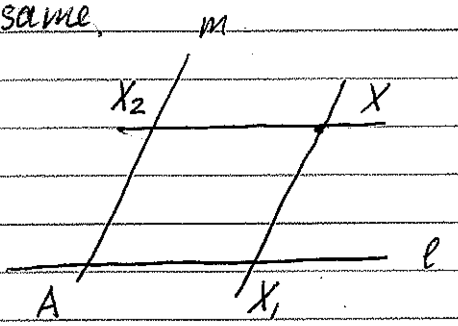
For any $C \in l$ let $CD \parallel XY$

CD meet m by (P) in C'

$C \to C'$ is a bijection between points in l and m.

For parallel l and m the same.

(b)



Consider again l and m meeting in A

Let X be a point consider
$m' \parallel m$ through X
it meets l in X_1 (by P')
$l'' \parallel l$ through X
it meets m in X_2

~~So~~ We have map Points $\longrightarrow l \times m$
and inverse map. Given $X_1 \in l$ and $X_2 \in m$
construct $m' \parallel m$ through X_1, $l' \parallel l$ through X_2.
They should meet by P in X.
Since $l \times m$ has n^2 elements, the statement follows.

(c) To check (P') let l is given by equation
$ax + by + c = 0$. Let A be the point with coordinates
(a_1, a_2)
Then $m \parallel l$ through A must be given by equation
$ax + by + d = 0$ So we find $d = -aa_1 - ba_2$.

(d) For 4, 9, 25 just take $\mathbb{Z}_2^2, \mathbb{Z}_3^2, \mathbb{Z}_5^2$.
$\mathbb{Z}_2, \mathbb{Z}_3, \mathbb{Z}_5$ are field with 2, 3 and 5 elements.
For 16 points we need the field F with 4 elements.
$\{0, 1, \cancel{x}, 1+x\}$ $x^2 = x + 1$ Check that this is a field.

Then consider F^2. It has 16 points.