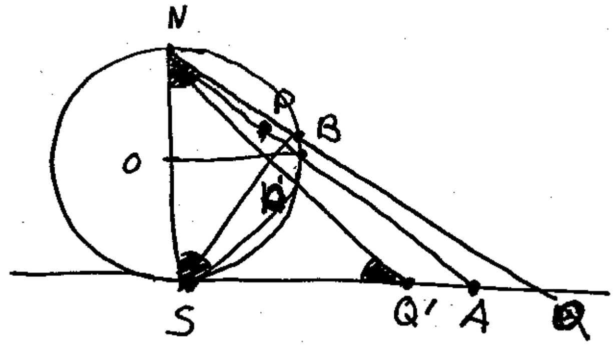
37.1

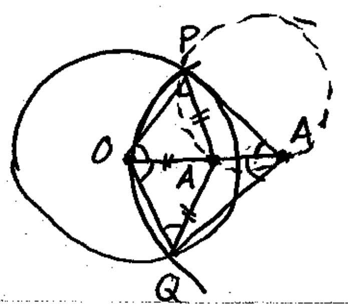


$\triangle NPS \cong \triangle SP'N$, $\angle NQ'S \cong \angle NSP' \cong \angle SNP$

$\triangle NSQ$ and $\triangle Q'SN$ are similar. So $\dfrac{NS}{SQ'} = \dfrac{SQ}{NS}$.

So $NS^2 = SQ' \times SQ$, But $NS = SA = 2r$. Thus, $SA^2 = SQ \times SQ'$

37.2



$\angle AOP \cong \angle OPA$

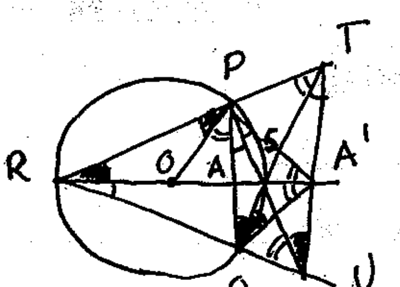
$\angle AOP \cong \angle PA'O$

The circle PAA' is tangent to OP. (III.32 invers

So $OP^2 = OA \times OA'$ (III.36)

T

37.11



UP and TQ are altitudes of $\triangle RTU$. So RS is also an altitude of $\triangle RTU$.

$\angle SPA' \cong \angle STA' \cong \angle SRU \cong \angle QPS = \beta$

$\angle ARP \cong \angle PQS \cong \angle PUT \cong \angle SQA' = \alpha$

$\angle PTS \cong \angle PA'S \cong \angle PUQ \cong \angle SA'Q = \gamma$

$OR \cong OP \Rightarrow \angle OPR = \alpha$

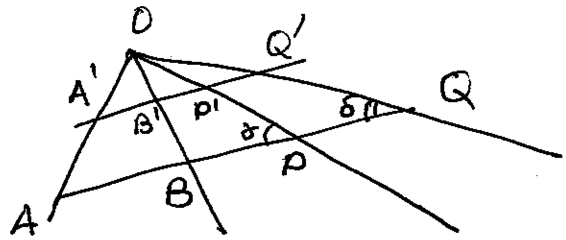
$\alpha + \beta + \gamma = RA \Rightarrow \angle OPA = \gamma = \angle OA'P$

The circle PAA' is tangent to OP.

We obtain $OP^2 = OA \times OA'$

37.14 (a)

$$(AB, PQ) = \frac{AP}{AQ} \div \frac{BP}{BQ} =$$



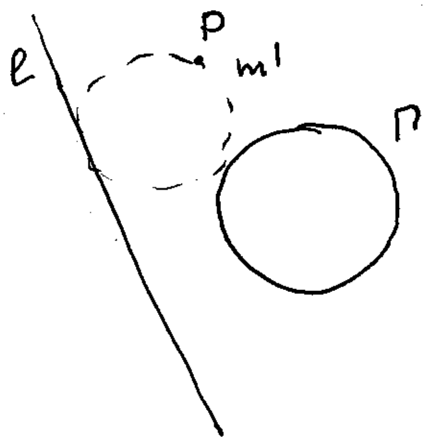
$$\frac{AP}{\sin \alpha_P} = \frac{OA}{\sin \gamma} \qquad \frac{BP}{\sin \beta_P} = \frac{OB}{\sin \gamma}$$

$$\frac{AQ}{\sin \alpha_Q} = \frac{OA}{\sin \delta} \qquad \frac{BQ}{\sin \beta_Q} = \frac{OB}{\sin \delta}$$

$$(AB, PQ) = \frac{\cancel{\sin \delta}}{\cancel{\sin \gamma}} \cdot \frac{\sin \alpha_P}{\sin \alpha_Q} \cdot \frac{\sin \beta_Q}{\cancel{\sin \delta}} \cdot \frac{\cancel{\sin \gamma}}{\sin \beta_P} = \frac{\sin \alpha_P}{\sin \alpha_Q} \div \frac{\sin \beta_P}{\sin \beta_Q}$$

(b) Follows from A since $\alpha_P, \alpha_Q, \beta_P, \beta_Q$ coincide with $\alpha_{P'}, \alpha_{Q'}, \beta_{P'}, \beta_{Q'}$.

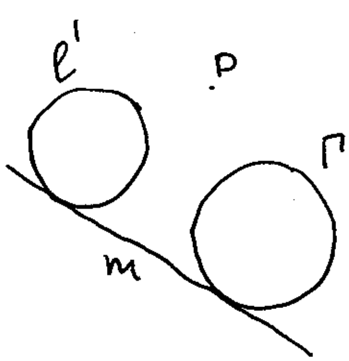
38.1



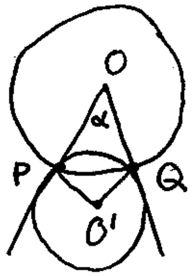
Let S be some inversion with center P

$S(\ell) = \ell'$
$S(\Gamma) = \Gamma'$
ℓ' and Γ' are circles.

Construct a line m tangent to Γ' and ℓ' and invert back to the circle m'



39.3 Without loss of generality we may assume that the vertex of α is the center of Γ



Let $\alpha = \angle POQ$

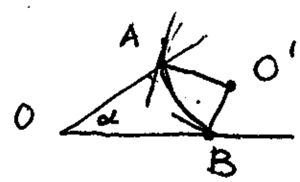
$O'Q$ is perpendicular OQ and $O'P \perp OP$.

OO' is the angle bisector of α.

The circle γ through P and Q is orthogonal to Γ by construction.

The P-line PQ is inside α.

39.6.



Have to construct O' such that $\angle OAO' = RA + \beta$

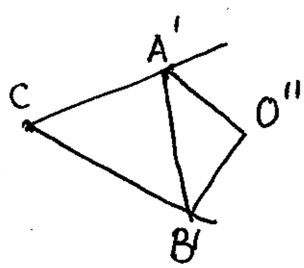
$\angle OBO' = RA + \gamma$

Note that the angle $\angle AO'B = 2RA - \alpha - \beta - \gamma$.

Start with constructing an isosceles triangle $A'O''B'$ with $\angle A'O''B' = 2RA - \alpha - \beta - \gamma$.

Draw. CA' and CB' so that $\angle CA'O'' = RA + \beta$

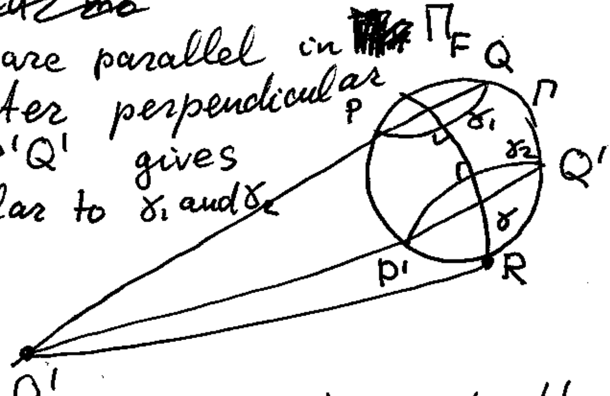
$\angle CB'O'' = RA + \gamma$



Using rigid motion move C to O and the use dilation to move A' to A and B' to B.

39.8. ~~Using rigid mo~~

If PQ and $P'Q'$ are parallel in Π then the diameter perpendicular to both PQ and $P'Q'$ gives P-line perpendicular to γ_1 and γ_2



Otherwise let PQ and $P'Q'$ meet at O'.

Let $O'R$ be tangent to and γ be the circle centered at O' and with radius $O'R$. Then $Q = S_\gamma(P)$, $Q' = S_\gamma(P')$ and $\gamma \perp \gamma_1$ and γ_2.