29.2.

(a) $\zeta+\zeta^{-1}+(\zeta+\zeta^{-1})^2+(\zeta+\zeta^{-1})^3-3(\zeta+\zeta^{-1})-1=$

$=1+\zeta+\zeta^{-1}+\zeta^2+\zeta^{-2}+\zeta^3+\zeta^{-3}=0$

$\alpha^3+\alpha^2-2\alpha-1=0$

$p(x)=x^3+x^2-2x-1$ is the minimal polynomial for α.

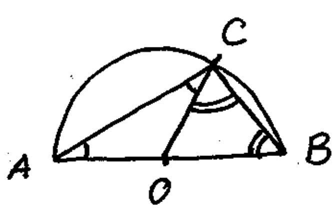
(b) The Galois group G of $\mathbb{Q}(\zeta)$ over $\mathbb{Q}$ is isomorphic to $\mathbb{Z}_6$. It has a unique subgroup of index 2. It is generated by $\zeta\mapsto\zeta^2$. The numbers $\beta=\zeta+\zeta^2+\zeta^4$ and $\beta'=\zeta^{-1}+\zeta^{-2}+\zeta^3$ are fixed by this subgroup. $\beta+\beta'=-1$, $\beta\beta'=2$

β,β' are roots of $x^2+x+2=0$

$$\beta,\beta'=\frac{-1\pm\sqrt{-7}}{2}$$

$d=-7$.

34.3



$\angle CAB=\angle ACO=\alpha$

$\angle OCB=\angle CBA=\beta$

$\delta(\triangle ABC)=2RA-2\alpha+2\beta=2RA-2\angle ACB$

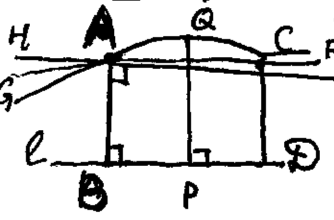
If Π is semielliptic $2RA-2\angle ACB<0\Rightarrow\angle ACB>RA$

If Π is semihyperbolic $2RA-2\angle ACB>0\Rightarrow\angle ACB<RA$

If Π is semiEuclidean $2RA=2\angle ACB\Rightarrow\angle ACB=RA$

Let $\ell=BD$, $AB\perp BD$.

34.5 Consider a Saccheri quadrilateral (ABCD)! Assume that the plane is semielliptic.



The line $PQ\perp BD$, $PQ\perp AC$. So $BD\parallel AC$

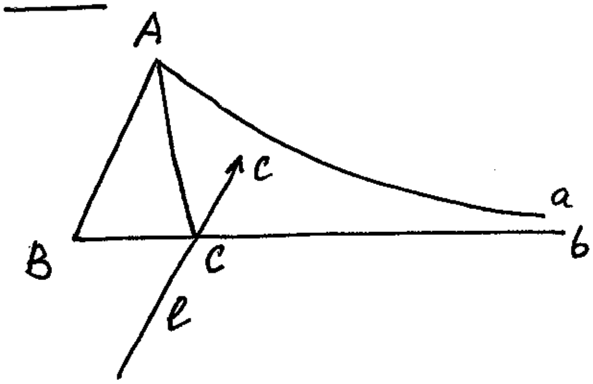
Let $AF\perp AB$. Then $AF\parallel BD$

Let AF_1 be the angle bisector of $\angle CAF$, AF_2 be the angle bisector of $\angle CAF_2$, ... e.t.c.

Eeach line $\angle AF_n$ lies inside $\angle CAF$ and $\angle HAG$, hence $AF_n\parallel BD$

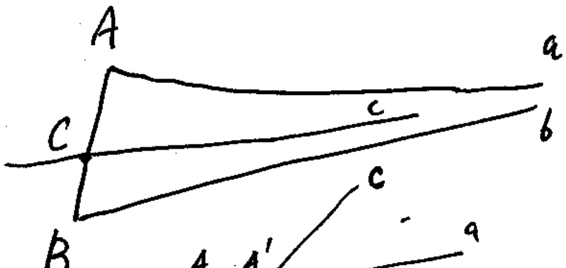
Similarly for semihyperbolic plane.

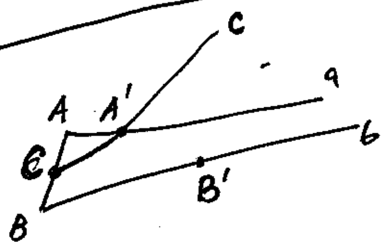
34.12.



Suppose l meets Bb in C, Let Cc lies on the same side of the line Bb as A. Since Cc does not contain A, then either Cc lies inside $\angle BCA$ and meets BA by the crossbar theorem, or Cc lies inside $\angle ACb$, hence Cc must meet Aa because $Cb \parallel\!| Aa$. It is also clear that Cc could not meet both AB and Aa, since in this case it must lie inside of $\angle BCA$ and $\angle ACb$, which is impossible.

Suppose l meets AB in C.

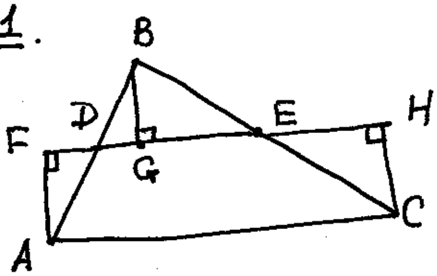




Let Cc be the ray of l on the same side of AB as Aa and Bb.
Assume that Cc does not meet Aa and Bb.
Then Cc is entirely in the interior of $\angle BAa$ and $\angle ABb$.
That implies $Cc \parallel\!| Aa$ and $Cc \parallel\!| Bb$.
(see Remark 34.12.1).

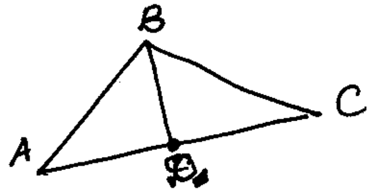
So if l does not contain a ray limiting parallel to Aa and Bb, then Cc must meet Aa or Bb.
Now we prove that Cc meets only one of Aa and Bb.
Indeed, assume that Cc meets Aa in A' and Bb in B'.
Assume without loss of generality that $C * A' * B'$.
Then B' and C lie on opposite sides of the line Aa.
~~Then $A'c$ lies on the opposite side of Aa then Bb~~
But C, B and B' lie on the same side of Aa.
Contradiction.

34.21.



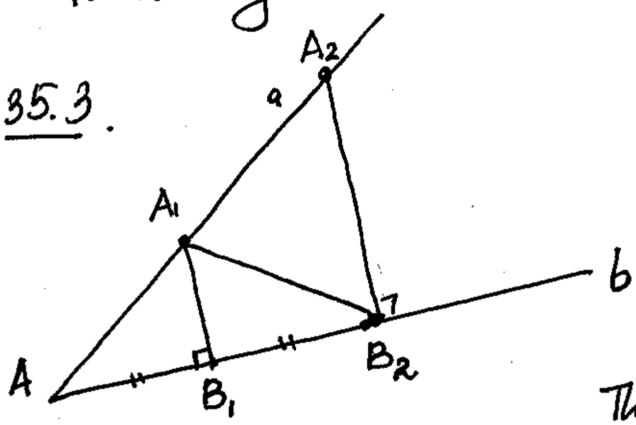
Construct the Saccheri quadrilateral as in the proof of Proposition 34.6.
The perpendicular bisector to AC is also perpendicular to $FH = DC$ by Proposition 34.1.

35.1. Let $\delta(ABC) = \delta$



Let D be the midpoint of AC.
Then $\delta(ABC) = \delta(ABD) + \delta(BDC)$.
Hence either $\delta(ABD) \le \frac{\delta}{2}$ or $\delta(BDC) \le \frac{\delta}{2}$.
Repeating this n times we can construct a triangle with defect less or equal to $\frac{\delta}{2^n}$.
Choose n such that $\frac{\delta}{2^n} < \varepsilon$, which is possible to do by (A).

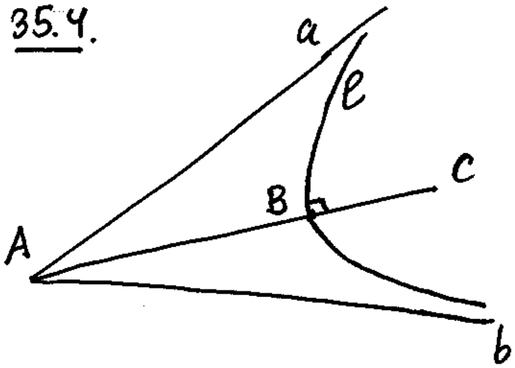
35.3.



If $\angle aAb \geq RA$ the statement is true for any point B.
Assume $\angle aAb < RA$.
Assume that the statement is false. Let $B_1 \in Ab$. ~~and~~
Then the perpendicular to Ab through B_1 meets Aa at A_1.

Let $\delta = \delta(AB_1A_1)$, and $B_2 \in Ab$ such that $B_1A \cong B_1B_2$. Then the perpendicular to B_2 meets Aa at A_2. $\delta(AA_2B_2) = \delta(AA_1B_1) + \delta(A_1B_1B_2) + \delta(A_1B_2A_2) > 2\delta$.
Repeating this process we can obtain a triangle AA_nB_n with defect greater than $2^n\delta$.
Since $\delta(AA_nB_n) < 2RA$, we obtain a contradiction with (A).

35.4.



let Ac be the angle bisector of $\angle aAb$.
By 35.3 there exists B on Ac such that the perpendicular l to AC does not meet Aa.
Then it does not meet Ab as well, since the reflection about AC maps l to itself and Aa to Ab.