SAMPLE MIDTERM
MATH 130

Midterm will cover Sections 6,7,8,9,10,11. This will be a closed book exam.

1. Let Π be an **incidence plane** that satisfies the conditions

• Every line contains exactly 7 points.

• Every point is contained in exactly 7 lines.

(a) Prove that every two lines meet at a point.

(b) Show that Π has 43 points.

2. In a Hilbert plane $A * B * C$ on one line and $A * D * E$ on another line. Show that the segment BE meets the segment CD.

3. Show that in any Hilbert plane there exists a quadrilateral with all sides congruent and all angles congruent.

4. Prove that in a Hilbert plane any triangle has at most one angle which is bigger than the right angle.

Solutions.

1. (a) Suppose that there are two lines l and m which do not intersect. Pick up a point A on l. Then for any point X on m there is a line AX. Thus, we have seven lines $AX_1, \dots, AX_7$ for each point X_i on m. In addition we have the line l. Therefore A is contained in at least 8 lines. Contradiction.

(b) Let m be a line consisting of points $X_1, \dots, X_7$. Let A be some point not on m. Since A is contained in exactly 7 lines, a line through A must coincide with AX_i for some i. Any point not equal to A must belong to one of these seven lines, and each of these lines has six points not counting A. Hence altogether we have $7 \times 6 + 1 = 43$ points.

2. Apply B4 to the triangle ABE and the line DC. DC must intersect AB or BE, but it could not intersect AB because C is not between A and B. Hence the line CD intersects the segment BE. By the same argument the line BE meets the segment CD.

3. Consider a line l. Take a point A on it. Pick up another point B on l and C on the opposite side such that $AB \simeq AC$ (C1). Let m be the line through A perpendicular to l, and D and E on m such that $AC \simeq AD \simeq AE$. Then the triangles BAD, CAD, BAE and CAE are congruent by C6. Therefore $BD \simeq DC \simeq CE \simeq EB$ and the angles are congruent by addition of angles.

4. Consider a triangle ABC. Assume that angles ABC and BAC are both bigger than the right angle. The supplementary to ABC is less than RA but it is bigger than BAC. Contradiction.