SAMPLE MIDTERM
MATH 130

Midterm will be on Friday September 26 during usual class hours. It will consist of 3 problems and cover the first chapter of Hartshorne book. You can use Euclid's books during the exam. You can refer to any statement in the first four books of Euclid or any proposition proved in Hartshorne. Below is a sample midterm (it has 4 problems since it was longer).

1. Using ruler and compass for a given point P inside an angle construct points A and B on the sides of an angle such that P is the midpoint of the segment AB. Count the number of steps and justify your construction.

2. Let ABCD be a cyclic quadrilateral such that $AB \cong CD$. Show that the sides BC and AD are parallel. (You may use any proposition from Book $1-4$ of Euclid.)

3. Using ruler and compass divide a given segment into 3 equal pieces.

4. Prove that if two medians of a triangle are equal, then the triangle is isosceles.

Date: September 17, 2014.

Solutions.

1. Denote the angle EDF. Here are the steps of construction

(1) line DP;
(2) a point C on DP such that $CP \cong PD$;
(3) line EP;
(4) a point G on EP such that $EP \cong PG$;
(5) line CG, the point $B = CG \cap DF$
(6) line BP, the point $A = BP \cap DE$.

Triangles DPE and CPG are congruent (by SAS, vertical angles DPE and CPG are congruent, $EP \cong PG$, $DP \cong PC$). Therefore the angles EDP and PCG are congruent. Then the triangles BPC and APD are congruent (by ASA we have the angles APD and BPC are vertical, $DP \cong PC$, and the angles EDP and PCG are congruent). Thus, $AP \cong PB$.

2. Let O be the point of intersection of the diagonals. By proposition 3.21 the angles ABO and OCD are congruent and the angles BAO and ODC are congruent. Then since $AB \cong CD$ the triangles ABO and DCO are congruent by ASA. In particular $OB \cong OC$ and $OD \cong OA$. Then the angles OBC and OCB are congruent and therefore the angles ABC and BCD are congruent. Since ABC and CDA together are two right angles by 3.23, we obtain BCD and CDA together are two right angles. Proposition 1.28 implies that BC is parallel to AD.

3. Given a segment AB. Draw a line AP and construct points C, D and E on AP so that $AC = CD = DE$. Draw the line EB and lines l through D and m through C parallel to EB. Let F be the point of intersection of l and AB and G be the point of intersection of m and AB. Then $AG = GF = FB$. For the proof use Thales theorem (see Exercise 5.3).

4. Consider a triangle ABC and medians AD and CE so that D is the midpoint of BC and E is the midpoint of AB. Given $AD = CE$. Let P be the point of intersection of AD an CE. It divides both medians in the ratio $2 : 1$. So we have $AP = PC$ and $EP = PD$. Furthermore, the angle EPA and CPD are congruent as vertical angles. Hence the triangles APE and CPD are congruent. That implies $AE = CD$ and hence $AB = 2AE = 2CD = BD$.