SPRING 2009. FINAL EXAM
MATH 130

Your name

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Student ID number

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For full credit it is sufficient to solve **6 problems**. You can try to do more problems but please note that there will be no partial credit for extra problems.

1	2	3	4	5	6	7	8	9	total

1. (a) Let $ABCD$ be a quadrilateral in a Hilbert plane such that the opposite sides are congruent. Prove that the opposite angles of $ABCD$ are also congruent.

(b) Prove that if the opposite angles of a quadrilateral are congruent, then the opposite sides are also congruent.

Warning: (P) does not hold in a Hilbert plane.

2. Let ABC be a triangle in an Euclidean plane and $AB \neq BC$. Let the angle bisector of the angle ABC and the perpendicular bisector to the side AC meet in M. Prove that M lies on the circle passing through A, B, C.

3. (a) Given an acute angle α in a Hilbert plane, prove that there exists a triangle with two angles congruent to α.

(b) Give a construction of such triangle by Hilbert tools.

4. Let $F = \mathbb{Q}(\sqrt{5})$ and Π_F be the Cartesian plane over F. Which of the congruence axioms (C1)-(C6) hold in Π_F ?

5. Given a P-angle α in a Poincare plane, give a ruler and compass construction (in the ambient Euclidean plane) of the P-line limiting parallel to both sides of α.

6. Consider a limit triangle in a hyperbolic plane with vertices A and B and angles α and β. Let C be a point inside this triangle. Prove that the defect of ABC is less than $2RA - \alpha - \beta$.

7. Let a be a real root of the polynomial $x^4 - 5x^2 + 1$.

(a) Is a constructible by ruler and compass?

(b) Is a constructible by Hilbert tools?

8. A rigid motion T in a Hilbert plane is called a translation if for any two points A and B the segments $AT(A)$ and $BT(B)$ are congruent. Prove that in a semihyperbolic or a semielliptic plane the only translation is the identity map.

9. Given a triangle in a Hilbert plane and a point M outside the triangle, prove that there exists a line l passing through M which does not intersect the sides of the triangle.