SPRING 2009. FINAL EXAM
MATH 130

1. Let ABC be a triangle in a Hilbert plane and M be the midpoint of BC. Prove that $2AM < AB + AC$.

2. Let $ABCD$ be a cyclic quadrilateral in an Euclidean plane and let the angle bisectors of the angles ABD and ACD meet in M. Prove that M lies on the circle passing through A, B, C, D.

3. (a) Given two acute angles α and β in a Hilbert plane, prove that there exists a triangle with angles α and β.

(b) Give a construction of such triangle by Hilbert tools.

4. Let $\Pi_{\mathbb{Q}}$ be the Cartesian plane over the field of rational numbers. Which of the congruence axioms (C1)-(C6) hold in $\Pi_{\mathbb{Q}}$?

5. Let A be a P-point and l be a P-line in a Poincare plane. Give a ruler and compass construction (in the ambient Euclidean plane) of a P-line limiting parallel to l and passing through A.

6. Let l and m be two distinct lines in a hyperbolic plane which are parallel but not limiting parallel. Prove that there exists a unique reflection r such that $m = r(l)$.

7. Let a be a real root of the polynomial $x^4 + 5x^2 - 1$.

(a) Is a constructible by ruler and compass?

(b) Is a constructible by Hilbert tools?

8. Let ABC be a triangle in a Hilbert plane, M be the midpoint of AB and N be the midpoint of AC. Show that if $BC = 2MN$, then the plane is semi-Euclidean. (Hint: construct a Saccheri quadrilateral with defect $\delta(ABC)$ and prove that it is a rectangle.)

9. Let AB be a P-segment in a Poincare plane and $\alpha(AB)$ be its angle of parallelism. Show that

$$\tan \alpha = \frac{2\mu(AB)}{\mu^2(AB) - 1},$$

where $\mu(AB)$ is the multiplicative distance.