\[ d^2(AB) = (1+m^2)(b_i-a_i)^2 \]
\[ d^2(BC) = (1+m^2)(c_i-b_i)^2 \]
\[ d^2(AC) = (1+m^2)(c_i-a_i)^2 \]

\[ (1+m_{i1}^2)(b_i'-a_i') = (1+m^2)(b_i-a_i)^2 \]
\[ (1+m_{i2}^2)(c_i'-b_i') = (1+m^2)(c_i-b_i)^2 \]

\[ \frac{b_i'-a_i'}{b_i-a_i} = \frac{c_i'-b_i'}{c_i-b_i} = \alpha \]
\[ \alpha^2 = \frac{1+m_{i1}^2}{1+m_{i2}^2} \]

\[ \frac{c_i'-a_i'}{c_i-a_i} = \frac{c_i'-b_i'+b_i'-a_i'}{c_i-b_i+b_i-a_i} = \alpha \]

We have \[ (1+m^2)(c_i'-a_i')^2 = (1+m^2)(c_i-a_i)^2 \]
\[ d^2(AC) = d^2(A'C') \]
16.6. Using rigid motions make the center of a circle to the origin and line to a vertical line.

Then the equation of the line is \( x = a \)
and the equation of the circle is \( x^2 + y^2 = r^2 \).

The line meets the circle in two points if \( y^2 = r^2 - a^2 \) has two solutions.
That happens if \( r^2 - a^2 > 0 \) or \( r^2 > a^2 \).
If \( r^2 > a^2 \), the point \( (a, 0) \) is on a line and inside the circle. If \( r^2 \leq a^2 \), the closest point to the center has coordinates \( (a, 0) \) and is outside or on the circle.

16.9. \( \cos 72^\circ = \frac{\sqrt{5} - 1}{4} \), \( \sqrt{5} = \sqrt{1 + 2^2} \in \mathbb{R} \)

\( \sin 72^\circ = \sqrt{1 - \left( \frac{\sqrt{5} - 1}{2} \right)^2} = \)

\( = \sqrt{\frac{16 - 5 - 1 + 2\sqrt{5}}{16}} = \sqrt{\frac{10 + 2\sqrt{5}}{4}} = \sqrt{\frac{(1 + \sqrt{5})^2 + 2^2}{4}} = \sqrt{\frac{1 + 2\sqrt{5} + (1 + \sqrt{5})^2}{4}} \in \Omega \)

since \( 1 + \sqrt{5} \in \Omega \)
17.3 (a) $\Phi(A) = A$
$\Phi(B) = B$
$\Phi(C) = C$

Let $D' = \Phi(B)$
then $\angle DAB \cong \angle D'B'AB$ and $DA \approx D'A$

In addition $(D$ and $C)$ and $(C$ and $D')$
are on the same side of $AB$.
Hence $D$ and $D'$ are on the same side.

and by $(C1)(C4) = D = D'$
Let $\Phi(\triangle ABC) = \triangle A'B'C'$

(b) First a reflection $r$, in

perpendicular bisector

$\Phi r_1 (A') = A'$

Let $\Phi r_1 (B) = B''$

Consider the reflection in the angle bisector
do $\angle B'A'B''$ moves $B'$ to $B''$

$r_2 \circ \Phi r_1 (A') = A'$

$r_2 \circ \Phi r_1 (B) = B'$

$r_2 \circ \Phi r_1 = \text{id}$ or $r_3 \circ \Phi r_1$ is the reflection

$r_3$ on line $A'B'$. Then we have

$\Phi = r_2 \circ r_4$

$\Phi = r_2 \circ r_3 \circ r_4$
17.5 (a) Construct \( \Phi \) as follows.

\[
\Phi(B) = B'
\]
so that \( BB' \parallel AA' \)
and \( AB \parallel A'B' \)

By \( \text{SAS} \) \( \Phi \) preserves
congruence of segment.
Hence \( \Phi \) is a rigid
motion.

(b) \( AA' \approx BB' \) (condition)
\( AB \approx A'B' \) (rigid motion)
\( ABB'A' \) is a parallelogram
Moreover, \( AB \) does not meet \( A'B' \) otherwise
the intersection point is fixed by translation
which is impossible.

Impossible case

\[
\begin{array}{c}
A \\
A' \\
A'' \\
\end{array}
\quad \quad \quad \quad \quad
\begin{array}{c}
T_1 \\
T_2 \\
B \\
\end{array}
\]

\[
\begin{array}{c}
T_1(A) = A' \\
T_2(B') = B'' \\
\end{array}
\]

By \( \text{SAS} \) \( AA'' \approx BB'' \).
Hence \( T_2 \circ T_1 \) is again translation.

(c) Yes. Let \( \Phi \) be any rigid motion,
\( T \) be a translation

Let \( \Phi \circ T \circ \Phi^{-1}(A) = A' \), \( \Phi \circ T \circ \Phi^{-1}(B) = B' \)
Let \( A = \Phi(A'') \), \( B = \Phi(B'') \)
Then \( \Phi \circ T \circ \Phi^{-1}(A) = \Phi \circ T(A'') \)
Yes.

17.5(d) Let $T$ be a translation and $\phi$ be a rigid motion.
We have to show that $\phi \circ T \circ \phi^{-1}$ is a translation. Let $A' = \phi \circ T \circ \phi^{-1}(A)$
and $B' = \phi \circ T \circ \phi^{-1}(B)$.

Let $A'' = T \circ \phi^{-1}(A)$, $B'' = T \circ \phi^{-1}(B)$.
Then $A''' = \phi^{-1}(A)$, $B''' = \phi^{-1}(B)$.

Since $T$ is a translation,

$A'' A'''' \cong B'' B''''$

Since $\phi$ is a rigid motion

$A' A \cong B' B$

Therefore $\phi \circ T \circ \phi^{-1}$ is a translation by definition in (a).

17.10 Solution repeats 17.3(b).

17.11 If $S_1$ is a rotation on angle $\alpha_1$ and $S_2$ is a rotation on angle $\alpha_2$, for any
line $l$, $S_2(l)$ and $l$ has angle $\alpha_1$
and $S_2 S_1(l)$ and $S_1(l)$ has angle $\alpha_1 + \alpha_2$.
Thus, if $S = S_2 \circ S_1$, and $A$ and $B$ are
fixed by $S$, then the whole line $AB$ is fixed
but then $S$ is a reflection. That is impossible
because the angle between $l$ and $S(l)$
depends on $l$. So if $S$ has a fixed point,
then $S$ is a rotation.

Assume that $S$ does not have fixed
points.
Now consider the case when $g$ does not have fixed points. Let $g(A) = A'$ and $g(A') = A''$.

Note that $A'' \neq A$ (otherwise midpoint of $AA'$ is fixed). If $A, A'$ and $A''$ are not collinear, then there is a unique $O$ such that $OA = OA' = OA''$.

Then $g(O) = O$, or $g(O) = O'$, where $O'$ is a point symmetric to $O$ with respect to $AA''$.

But the latter case is impossible since then $OA' \parallel O'A''$ but $AA'$ is not parallel to $A'A''$ and we proved before that the angle between $l$ and $g(l)$ does not depend on $l$.

On the other hand, $g(O) = O$ also impossible since $g$ does not have fixed points.

That proves $A, A', A''$ are collinear.

Take another point $B$, let $B' = g(B)$.

Then $AA' \parallel B'B$ (otherwise the point of intersection is fixed), $<BAA' \cong B'A'A''$ so $AB \parallel A'B'$. Then $AA' \sim BB'$

and that show $T$ is a translation.

17.18. Note that the composition of a translation and a rotation is a translation (proof as in 17.11).

The composition of two reflections is either a rotation or a translation. So $r_1 r_2 \ldots r_k x_1 \cdots x_l$ implies $x_1 \cdots x_k x_1' \cdots x_l' = x_1' \cdots x_l'$, $LHS$ is a rotation or a translation. $RHS$ is contradiction.