3.4
1) \( DE = CD \)
2) \( O \) the midpoint of \( AE \)
3) \( O \equiv 0 \equiv OE \), get \( M \) on \( DN \)
4) \( ND \equiv DM (\therefore) \)
   \( \square \) with side \( DN \)
   \( DN \times DM = AB \times DE \)
   \( DN^2 = AB \times BC \) (\( III \ 35 \))

3.5
I. If \( AB \) is not parallel to \( l \), get \( P = (AB) \cap l \)
   Construct \( PO \) such that \( PO^2 = PA \times PB \) (as in 3.4)
   Draw a circle around \( ABO \) (\( III \ 37 \))
II. If \( AB \) is parallel to \( l \), draw perpendicular bisector \( OB \) to \( AB \), get \( O \)
   Draw a circle around \( ABO \) (\( III \ 3 \))

3.10
\( \text{Cont} (AFD) = \text{Cont} (AED) \) (I.38)
\( \text{Cont} (ABD) = \text{Cont} (AEC) = \frac{1}{2} \text{Cont} (ABC) \) (I.38)
\( \text{Cont} (AFDB) = \text{Cont} (AEB) - \text{Cont} (ADE) + \text{Cont} (AFD) = \text{Cont} (AEB) = \frac{1}{2} \text{Cont} (ABC) \)
4.7  \( EF \times GE = AE \times EC \) (III. 35)

\[ GE \cong AE \cong EC \] since \( \triangle ADE \) is equilateral

\( BE \) being median (all angles congruent)

\( PF \cong PG \) (III. 3)

\( BP \cong PE \) (\( \triangle ADE \) isosceles)

So \( \angle PAB \cong \angle PEB \) and \( GE \parallel DF \)

By (1) \( EF \times DF = DE^2 \)

5.13

\( \angle DPE \) is cyclic \( \Rightarrow \)

\( \angle EBC \cong \angle EPC \)

\( \angle FBD \) is cyclic \( \Rightarrow \)

\( \angle DBF \cong \angle BPF \)

\( \angle BAF \) is cyclic \( \Rightarrow \angle BAC + \angle FPE = 2\alpha \)

\( \angle APB \) is cyclic \( \Rightarrow \angle BAC + \angle BPC = 2\alpha \)

Therefore \( \angle FPE \cong \angle BPC \Rightarrow \angle BPE \cong \angle EPC \)

Thus, \( \angle DBF \cong \angle EBC \) \( \cong \) vertical \( \Rightarrow \)

\( EBF \) on the same line.
5.15

By construction, \(AB'C'\) is cyclic. Hence, \(\angle BCA = \angle DB'A\).

But \(AB' \parallel BA'\), therefore \(\angle DB'A = \angle DA'B\). So \(\angle BCA = \angle BA'B\).

Therefore, \(DB'CB\) is cyclic.

\[\angle CBA = \angle B'CA\] since \(B'C' \parallel CB\).

\[\angle ABB' + \angle BCA = 2RA\], since \(AB'CB\) is cyclic, and \(\angle C'BA + \angle ABB' = 2RA\). Therefore \(\angle C'BA = \angle CBA\) and \(AC'B'\) is cyclic.

Thus, we have \(\angle BBA' + \angle C'B'C = 2RA\), \(\angle BAC' + \angle BAC = 2RA\).

Therefore \(\angle C'AB + \angle BCA' = 2RA \Rightarrow C'A \parallel A'C\).

5.18

Assume the contrary. Pick up any point \(A\).

Let the color of \(A\) be blue.

\(\Delta ABC \cong \Delta BCD\), both equilateral.

Then \(A, B, C\) have 3 different colors.\[\text{and } B, C, D\] have 3 different colors.

Therefore \(D\) must be blue again.

All points on the circle centered at \(A\) and radius \(AB\) are blue.

Take the circle with center \(D\) and radius \(AB\). It meets \(\Gamma\) in two points \(F\) and \(E\).

\(F\) and \(E\) have the same color. \[DE = 1AB = 1\]