\[ \triangle NPS \cong \triangle SP'N, \quad \angle NQ'S \equiv \angle NSP' \equiv \angle SNP \]
\[ \triangle NSQ \text{ and } \triangle Q'SN \text{ are similar. So } \frac{NS}{SQ'} = \frac{SQ}{NS}. \]
So \[ NS^2 = SQ' \cdot SQ, \text{ But } NS = 5a = 2r. \text{ Thus, } SA^2 = SQ' \cdot SQ. \]

\[ \angle AOP \equiv \angle OPA \]
\[ \angle AOP \equiv \angle OPA' \]
The circle \( PA'A' \) is tangent to \( OP \). (III.32, invers)
So \[ OP^2 = OA \cdot OA' \] (III.36)

\[ \angle SPA' \equiv \angle STA' \equiv \angle SRA' \equiv \angle QPS = \beta \]
\[ \angle ARP \equiv \angle PQS \equiv \angle PUT \equiv \angle SQR' = \alpha \]
\[ \angle PTS \equiv \angle PA'S \equiv \angle PUQ \equiv \angle SA'Q = \gamma \]
\[ OR \equiv OP \Rightarrow \angle OPR = \alpha \]
\[ \alpha + \beta + \gamma = RA \Rightarrow \angle OPA = \gamma = \angle OA'P \]
The circle \( PA'A' \) is tangent to \( OP \).
We obtain \[ OP^2 = OA \cdot OA' \]
37.14. (a) 
\[ (AB, PQ) = \frac{AP}{AQ} : \frac{BP}{BQ} = \frac{OA}{sin \alpha_p} : \frac{OB}{sin \delta} \]

\[
\frac{AP}{sin \alpha_p} = \frac{OA}{sin \gamma} \\
\frac{AQ}{sin \delta} = \frac{OA}{sin \delta} \\
\frac{BP}{sin \beta_p} = \frac{OB}{sin \delta} \\
\frac{BQ}{sin \beta_q} = \frac{OB}{sin \delta}
\]

\[ (AB, PQ) = \frac{sin \delta}{sin \gamma} \cdot \frac{sin \alpha_p}{sin \delta} \cdot \frac{sin \beta_q}{sin \delta} \cdot \frac{sin \gamma}{sin \beta_p} \cdot \frac{sin \beta_p}{sin \delta} \cdot \frac{sin \beta_q}{sin \delta} \]

(b) Follows from A since \( \alpha_p, \alpha_q, \beta_p, \beta_q \) coincide with \( \alpha'_p, \alpha'_q, \beta'_p, \beta'_q \).

38.1. Let \( \mathcal{S} \) be some inversion with center \( P \).

\( \mathcal{S}(l) = l' \) and \( \Gamma' \) are circles.

\( \mathcal{S}(\Gamma) = \Gamma' \)

Construct a line \( m \) tangent to \( \Gamma' \) and \( l' \) and invert back to the circle \( m' \).
39.3 Without loss of generality we may assume that
the vertex of \( \alpha \) is the center of \( \Gamma \)

Let \( \alpha = \angle P Q O \)

- \( O'Q \) is perpendicular to \( OQ \)
- \( O'P \parallel OP \)

- \( O'O' \) is the angle bisector of \( \alpha \)

The circle \( \gamma \) through \( P \) and \( Q \)

is orthogonal to \( \Gamma \) by construction.

The \( P' \)-line \( PQ \) is inside \( \alpha \).

39.6 Have to construct \( O' \) such that
\[ \angle AAO' = RA + \beta \]
\[ \angle BO'O' = RA + \gamma \]

Note that the angle \( \angle AOB' = 2RA - \alpha - \beta - \gamma \).

Start with constructing an isosceles triangle
\( A'O'B' \) with
\[ \angle A'O'B' = 2RA - \alpha - \beta - \gamma \]

Draw \( CA' \) and \( CB' \) so that
\[ \angle CA'O'' = RA + \beta \]
\[ \angle CB'O'' = RA + \gamma \]

Using rigid motion move
\( C \) to \( O \) and the use
dilation to move \( A' \) to \( A \)
and \( B' \) to \( B \).

39.8 Using rigid zone

If \( PQ \) and \( P'Q' \) are parallel in \( \Pi \),
then the diameter perpendicular
to both \( PQ \) and \( P'Q' \) gives
\( P' \)-line perpendicular to \( \gamma \) and \( \delta \).

Otherwise let \( PQ \) and \( P'Q' \)
meet at \( O' \).

Let \( O'R \) be tangent to \( \Gamma \) and \( \delta \) be the circle centered
at \( O' \) and with radius \( O'R \). Then \( Q = Q(P), Q' = Q(P') \) and \( \delta \perp \gamma \) and \( \delta_2 \).