SAMPLE MIDTERM
MATH 130

Midterm will be on February 27 during usual class hours. It will cover Sections $1-6$ of the Hartshorne book. This will be an open book exam. You can refer to any statement in the first four books of Euclid or any proposition proved in Hartshorne.

1. Using ruler and compass for a given point P inside an angle construct points A and B on the sides of the angle such that P is the midpoint of the segment AB. Count the number of steps and justify your construction.

2. Let $ABCD$ be a cyclic quadrilateral such that $AB \cong CD$. Show that the sides BC and AD are parallel.

3. Prove that on a finite incidence plane satisfying the Playfair axiom two parallel lines have the same number of points.

4. Let ABC be a non- isosceles. Let M be the point of intersection of the angle bisector of the angle ABC and the perpendicular bisector to the side AC. Prove that M lies on the circle ABC.

5. Given a parallelogram $ABCD$, let P be the point of intersection of the diagonals. Prove that any line passing through P divides the parallelogram into two figures of equal content.

Date: February 16, 2009.

Solutions.

1. Denote the angle EDF. Draw the line DP, then draw the circle with radius DP and center P. Let C be the second point of the intersection of the circle and the line DP. Through C draw the line CH parallel to DF and CG parallel to ED (1.31). Let A be the point of intersection of CH and ED, B be the point of intersection of CG and DF. By construction $DBCA$ is a parallelogram, the diagonal AB passes through P, the midpoint of AB and DC.

2. The angles DAC and ACB are congruent as subtended by congruent chords AB and CD. Proposition 1.27 implies that BC is parallel to DA.

3. Let l and m be two parallel lines. If both lines contain only two points, there is nothing to prove. Otherwise we claim that there is a point A which does not lie on l or on m. Indeed, assume the opposite. One of the lines, say l, has at least three points B_1, B_2, B_3, and the other has at least two points by $(I2)$ $C_1$and C_2. Then the lines B_2C_2 and B_3C_2 are parallel to B_1C_1, that contradicts (P).

For each point B_i on m the line AB_i meets l at some point C_i, since AB_i can not be parallel to l by (P). By $(I1)$ if $B_i \neq B_j$, then $C_i \neq C_j$. Hence the number of points on l is not less than the number of points on m. But similarly the number of points on m is not less than the number of points on l. So m and l have the same number of points.

4. Let N be the point of intersection of the perpendicular bisector to AC and the circle ABC that lies on the opposite side of AC relative to B. Then $NA \cong NC$, hence the angles NBA and NBC are congruent. So N lies on the angle bisector of ABC. Therefore, $N = M$.

5. Let l be a line passing through P. Consider the case when l lies inside the angles APB and CPD. The other case is absolutely similar. Let F and G be the points of intersection of l and the sides AB and CD respectively. We have to show that the content of $BFGC$ and the content of $AFGD$ are equal. Note that the triangles ABC and CDA by ASA are congruent, therefore their content equals a half of the content of the parallelogram. The triangles APF and CPG are also congruent by ASA ($CP \cong PA$, the angle FAP is congruent to PCG and FPA is congruent GPC as vertical angles). So they also have equal contents. Thus, the content of $BFGC$ equals content of ABC minus content of AFP plus content of PGC equals half content of $ABCD$, the same for $AFGD$.