HOMEWORK 1 SOLUTIONS

1.15. The area is
\[ \frac{1}{4} + \frac{3}{16} + \frac{9}{64} + \cdots = \sum \frac{3^{n-1}}{4^n} = \frac{1}{4} \left( 1 - \frac{3}{4} \right) = 1 \]

7.10. The condition \(|a_n| > |a_{n+1}|\) does not hold, one can not apply the alternating series test.

9.6. Use the ratio test.
\[ \lim \left( \frac{(n+1)!}{(2n+2)!} / \frac{(n)!}{(2n)!} \right) = \lim \frac{(n+1)^2}{(2n+1)(2n+2)} = \frac{1}{4} \]
Hence the series is convergent.


9.19. The series is the difference of two convergent geometric series. Hence it is convergent.

14.5. Use the estimate for alternating series (14.3). The next term is
\[ \frac{x^4}{4!} \leq \frac{1}{2^4 \times 4!} < 0.003 \]

15.7.
\[ x^8 \tan^2 x = x^8 (x + \ldots)^2 = x^{10} + \ldots \]
Thus,
\[ \frac{d^{10}}{dx^{10}} (x^8 \tan^2 x) \big|_{x=0} = 10! \]

15.31. The total area of the disks
\[ \pi \sum \frac{1}{n^2 \ln^2 n} \]
is convergent by comparison with \( \sum \frac{1}{n^2} \). Hence the area is finite. The total length is infinite. It is given by the infinite series
\[ 2\pi \sum \frac{1}{n \ln n} \]
It is divergent by integral test (done in class). The explanation relates to the fact that in real life the wire has a certain thickness, so when this thickness is less than \( \frac{1}{n \ln n} \) one should stop.

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