Sample midterm.

1. Solve the equation $x^2 = 3$ in $\mathbb{Z}_{2007}$.

2. Evaluate 3^{198} in $\mathbb{Z}_{19}$.

3. Let R be a ring of four elements $\{a, b, c, d\}$ with addition and multiplication given by the tables

$+$	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	b
d	d	c	b	a

$\cdot$	a	b	c	d
a	a	a	a	a
b	a	b	a	b
c	a	a	c	c
d	a	b	c	d

(a) Is R commutative?

(b) Is R a ring with identity element?

(c) Is R a field?

4. List all units in the ring $\mathbb{Z}_9$. For each unit $a \in \mathbb{Z}_9$ find its multiplicative inverse a^{-1}.

5. Show that the ring $\mathbb{Z}_9$ is not isomorphic to the ring $\mathbb{Z}_3 \times \mathbb{Z}_3$.

6. Let R and S be commutative rings with identity. Prove or disprove the following statements.

a) If $f : R \to S$ is a surjective homomorphism of rings and S is an integral domain, then R is an integral domain.

b) If $f : R \to S$ is an injective homomorphism of rings and S is an integral domain, then R is an integral domain.