SAMPLE FINAL 2
MATH 113

1. Evaluate $2^{2007} \pmod{19}$.
2. Determine if the polynomial $x^5 + 3x + 3$ is irreducible
(a) Over $\mathbb{Q}$.
(b) Over $\mathbb{Z}_7$.
3. Let $R = \mathbb{Z}[x]$.
(a) Show that R is an integral domain.
(b) Find all units of R.
4. Let p be an odd prime number. Show that the equation

$$x^2 = -1$$

has a solution in $\mathbb{Z}_p$ if and only if $p \equiv 1 \pmod{4}$. (Hint: use the fact that the group of units is cyclic.)
5. Show that the groups D_6 and A_4 are not isomorphic.
6. Show that the quotient ring $\mathbb{Z}_{25}/(5)$ is isomorphic to $\mathbb{Z}_5$.
7. Show that the rings $\mathbb{Z}_{25}$ and $\mathbb{Z}_5[x]/(x^2)$ have the same number of elements but not isomorphic.
8. How many Sylow 5-subgroups does the group A_5 have? Write down one Sylow subgroup and its normalizer.
9. Show that every group of order 51 is cyclic.
10. Show that $\mathbb{Q}[x]/(x^2 + x + 1)$ and $\mathbb{Q}[x]/(x^2 + 3)$ are isomorphic. (Hint: show that $\mathbb{Q}[x]/(x^2 + 3)$ contains a root of $x^2 + x + 1$.)