

**SAMPLE FINAL**  
**MATH 113**

- 1.**
  - a) List all proper nontrivial subgroups in the group  $\mathbb{Z}_3 \times \mathbb{Z}_3$ ;
  - b) List all proper nontrivial ideals in the ring  $\mathbb{Z}_3 \times \mathbb{Z}_3$ .
- 2.** Let  $U_{10}$  be the group of units in the ring  $\mathbb{Z}_{10}$ . Show that  $U_{10}$  is isomorphic to  $\mathbb{Z}_4$ . List all generators of  $U_{10}$ .
- 3.** List all group homomorphisms
  - a) of  $\mathbb{Z}_6$  into  $\mathbb{Z}_3$ ;
  - b) of  $S_3$  into  $\mathbb{Z}_3$ .Explain your answer.
- 4.** Find all normal subgroups of  $S_4$ .
- 5.** Factor the polynomial  $x^2 + 3x - 1$  into a product of irreducibles in the ring
  - a)  $\mathbb{Q}[x]$ ;
  - b)  $\mathbb{Z}_{13}[x]$ .
- 6.** Let  $\phi_6: \mathbb{Z}_{11}[x] \rightarrow \mathbb{Z}_{11}$  be the evaluation homomorphism, given by  $\phi_6(p(x)) = p(6)$ .
  - a) Find  $\phi_6(x^{123} - x^{10} + 1)$ ;
  - b) Is  $\text{Ker}(\phi_6)$  a principal ideal? Explain your answer.
- 7.** Determine which of the following rings are integral domains:
  - a)  $\mathbb{Z}_{15}$ ;
  - b)  $\mathbb{Z} \times \mathbb{Z}_5$ ;
  - c)  $\mathbb{Z}_{11}[x]$ .
- 8.** Find the degree of  $\mathbb{Q}(\sqrt{3}, \sqrt[5]{7})$  over  $\mathbb{Q}$  and write down a basis of  $\mathbb{Q}(\sqrt{3}, \sqrt[5]{7})$  over  $\mathbb{Q}$ .
- 9.** Find the irreducible polynomial of  $\sqrt{5} + \sqrt{6}$  over  $\mathbb{Q}$  and prove that your answer is correct.
- 10.** Find an abelian subgroup of maximal order in  $S_5$ .