MATH 16B, SPRING 2004 SECOND MIDTERM SAMPLE

1. Compute the following indefinite integrals.

- $\int x \ln(7x) dx$
- $\int x \sin(x) dx$ $\int x \sin(x^2) dx$ $\int \frac{dx}{x(\ln(x))^5}$

2. Using Simpson's method with n = 2 subdivisions, compute an estimate for $\int_1^5 \frac{dx}{x}$.

3. Compute the following improper integrals. If the integrals fail to converge, say so.

• $\int_{-\infty}^{0} e^x \cos(e^x) dx$

•
$$\int_{-1}^{1} \frac{dx}{x^4}$$

•
$$\int_{-\infty}^{\infty} x dx$$

•
$$\int_0^2 \frac{\mathrm{In}(x)}{x} dx$$

4. Find the general solution to the differential equation $y' - y = e^t$.

5. Find the solution to the initial value problem $y' = ty^3 e^t$ and y(0) = -1.

6. It had been proposed that the velocity of an object in a free fall is proportional to the distance it has fallen. Let f(t) denote the distance fallen at time t.

- Write the differential equation satisfied by f.
- Find the general solution to this differential equation.
- In this case, this differential equation is implicitly an initial value problem. What is f(0)? What is the solution to this initial value problem? What is wrong with this mathematical model?

7. Compute the following definite integrals.

- $\int_0^1 x \sin(\pi x) dx$ $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot(x) dx$