MATH 16B, SPRING 2004
PRACTICE FOR THE FINAL EXAM

There are more questions listed here than you will actually see on the final. The *general* distribution of the questions accurately reflects the topics to be covered.

1. Find the point (x, y, z) which maximizes the function $F(x, y, z) = 3x + 5y + z - x^2 - y^2 - z^2$ subject to the constraint $g(x, y, z) = 6 - x - y - z = 0$.

2. Let R be the region bounded by $0 \leq x \leq 1$ and $e^x \leq y \leq 2e^x$. Compute

$$\iint_R y \sin(e^{2x}) dx dy$$

3. Compute $\int \ln(\sqrt[3]{x}) dx$

4. Compute $\int_2^3 \frac{dx}{x \ln(x)}$

5. Using the trapezoidal estimate and $n = 3$ subdivisions, compute an approximation to $\int_0^1 \frac{dx}{1+x^2}$

6. Compute $\int_0^\infty \frac{x}{x^2+1} dx$. If the integral diverges, say so.

7. Find the general solution to the differential equation $y' - t^3 y = 0$.

8. Find the general solution to the equation $y' + y = 2 - e^t$.

9. Sketch a graph of the solution to the initial value problem $y' = y^2 - y$ and $y(0) = \frac{3}{4}$.

10. Using Euler's method and $n = 2$ steps, compute an approximation to $y(2)$ where $y' = ty$ and $y(1) = 4$.

11. Using Newton's method, compute an approximation to $10^{\frac{2}{3}}$ valid to the tenths place.

12. Does the series $\sum_{n=1}^\infty \frac{1}{n2^n}$ converge? Justify your answer.

13. Find the Taylor series at $a = 0$ for the function $f(x) = x \int_0^x \frac{dt}{1+t^2}$.

14. Let $g(x) = x^2 e^{-x^3}$. Compute $g^{(8)}(0)$.

15. The scores on certain quiz were $7, 5, 10, 10, 8, 9, 2, 4$. What was the variance of this data set?

16. Let X be a continuous random variable with cumulative distribution function

$$F(x) = \begin{cases} 0 \text{ if } x < 0 \\ x^2 \text{ if } 0 \leq x \leq 1 \\ 1 \text{ if } x > 1 \end{cases}$$

Compute $E(X)$.

17. Suppose that the time between successive near Earth approaches by asteroids is exponentially distributed and that the average time between such passes is one hundred years. Suppose that an asteroid has just passed. What is probability that the next passing will be at least two hundred years from now? [Note the use of the word "suppose." This exercise does not reflect physical reality. However, the statement about a recent asteroid approach is correct: a large asteroid passed within one hundred thousand miles of the Earth in 2002.]

18. Suppose that the annual number of magnitude 7.0 or greater earthquakes recorded worldwide is Poisson distributed with a mean of 17. What is the probability of there being five or fewer such earthquakes next year?