Math 225A – Model Theory

Speirs, Martin

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General Information

These notes are based on a course in *Metamathematics* taught by Professor Thomas Scanlon at UC Berkeley in the Autumn of 2013. The course will focus on Model Theory and the course book is Hodges' *a shorter model theory.*

As with any such notes, these may contain errors and typos. I take full responsibility for such occurences. If you find any errors or typos (no matter how trivial!) please let me know at mps@berkeley.edu.

Lecture 11

Last time we proved that for finite signatures elementary equivalence is equivalent to $\exists$ having a winning strategy in the $\mathrm{EF}_k[\mathfrak{A}, \mathfrak{B}]$ for all $k < \omega$.

Corollary. *If τ has no relation symbols and $\mathfrak{A}_1, \mathfrak{A}_2, \mathfrak{B}_1$ and $\mathfrak{B}_2$ are τ-structures such that $\mathfrak{A}_1 \equiv \mathfrak{B}_1$ and $\mathfrak{A}_2 \equiv \mathfrak{B}_2$, then $\mathfrak{A}_1 \times \mathfrak{A}_2 \equiv \mathfrak{B}_1 \times \mathfrak{B}_2$.*

Proof. (Sketch) This basically follows from the fact that

$$\mathrm{EF}_k[\mathfrak{A}_1 \times \mathfrak{A}_2, \mathfrak{B}_1 \times \mathfrak{B}_2] = \mathrm{EF}_k[\mathfrak{A}_1, \mathfrak{B}_1] \times \mathrm{EF}_k[\mathfrak{A}_2, \mathfrak{B}_2].$$

(where the products of models is defined in the obvious way). $\square$

Hodges gives an application of this result to groups.

Corollary. *Let G_1, G_2 be elementarily equivalent groups, and let H be some group. Then $G_1 \times H$ is elementarily equivalent to $G_2 \times H$.*

The notes for this lecture are very short. This is because the material from the first part of lecture 11 was incorporated into the notes for lecture 10, and similarly the last part of lecture 11 was incorporated into the notes for lecture 12.