HOMEWORK ASSIGNMENT 9

Due in class on Wednesday, November 5.

Exercise 17.14 on page 125, Exercise 21.8 on page 163, plus Exercises AA–DD below.

AA. Find a one-to-one continuous function of \( \mathbb{R} \) onto \((0, 1)\).

BB. Let \( X, Y, Z \) be metric spaces. Let the functions \( f : X \to Y \) and \( g : Y \to Z \) be uniformly continuous. Prove \( g \circ f \) is uniformly continuous.

CC. Let \( (X, d) \) be a metric space. Regard \( X \times X \) as a metric space with the metric \( e \) defined by

\[
e((x_1, y_1), (x_2, y_2)) = d(x_1, x_2) + d(y_1, y_2).
\]

Prove the function \( d : X \times X \to \mathbb{R} \) is uniformly continuous.

DD. Let \( (X, d) \) be a metric space. Define \( e : X \times X \to \mathbb{R} \) by \( e(x, y) = d(x, y)/(1 + d(x, y)) \). Prove that \( e \) is a metric, that the identity map of \( (X, d) \) to \( (X, e) \) is uniformly continuous, and that the identity map of \( (X, e) \) to \( (X, d) \) is uniformly continuous.