HOMEWORK ASSIGNMENT 1

Due in class on Wednesday, September 8.

1. Prove that the number of subsets of the set \( \mathbb{N}_n = \{1, 2, \ldots, n\} \) is \( 2^n \).

2. Prove that, for \( m \) and \( n \) natural numbers, the number of ordered \( m \)-tuples whose coordinates belong to \( \mathbb{N}_n \) equals \( n^m \). (Suggestion: Use induction on \( m \).)

3. For \( n = 0, 1, 2, \ldots \) and \( k = 0, 1, \ldots, n \), the binomial coefficient \( \binom{n}{k} \) is defined by \( \binom{n}{k} = \frac{n!}{k!(n-k)!} \). (Recall the convention \( 0! = 1 \).)

   (a) Establish the identity \( \binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k} \).

   (b) Prove the binomial theorem:

   \[
   (x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k, \quad n = 1, 2, \ldots .
   \]