Problem Set 5

MATH 16B Spring 2016

10 March 2015

Exercise (9.3.5)**.** Compute
$$\int_0^3 \frac{x}{\sqrt{x+1}}dx.$$

Solution. Using the substitution $u = x+1$ (so $x = u-1$), we have $du = dx$ and $u = 1$ (resp. $u = 4$) when $x = 0$ (resp. $x = 3$), so
$$\begin{aligned}
\int_0^3 \frac{x}{\sqrt{x+1}}dx &= \int_1^4 \frac{u-1}{\sqrt{u}}du \\
&= \int_1^4 u^{1/2} - u^{-1/2}du \\
&= \left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right]_1^4 \\
&= \left(\frac{2}{3}\cdot 8 - 2\cdot 2\right) - \left(\frac{2}{3} - 2\right) \\
&= \frac{8}{3}.
\end{aligned}$$

□

Exercise (9.3.18)**.** Compute
$$\int_0^{\pi/4} \tan x\, dx.$$

Solution. Rewriting $\tan x$ as $\frac{\sin x}{\cos x}$, we can use the substitution $u = \cos x$, so that $du = -\sin x\, dx$ and $u = 1$ (resp. $u = \frac{\sqrt{2}}{2}$) when $x = 0$ (resp. $x = \frac{\pi}{4}$). So
$$\begin{aligned}
\int_0^{\pi/4} \tan x\, dx &= \int_0^{\pi/4} \frac{\sin x}{\cos x}dx \\
&= -\int_1^{\sqrt{2}/2} \frac{1}{u}du \\
&= -\Big[\ln u\Big]_1^{\sqrt{2}/2} \\
&= -\ln(\sqrt{2}/2) + \ln 1 \\
&= -\ln(\sqrt{2}/2).
\end{aligned}$$

□

Exercise (9.6.40)**.** Evaluate

$$\int_{-\infty}^{0} \frac{8}{(x-5)^2} dx.$$

Solution. Using the substitution $u = x - 5$, we have $du = dx$, and $u = -\infty$ (resp. $u = -5$) when $x = -\infty$ (resp. $x = 0$). Thus

$$\begin{aligned}
\int_{-\infty}^{0} \frac{8}{(x-5)^2} dx &= \int_{-\infty}^{-5} \frac{8}{u^2} du \\
&= \left[-\frac{8}{u} \right]_{-\infty}^{-5} \\
&= \frac{8}{5}.
\end{aligned}$$

□

Exercise (9.6.43)**.** Evaluate

$$\int_{0}^{\infty} \frac{e^{-x}}{(e^{-x}+2)^2} dx.$$

Solution. Using the substitution $u = e^{-x} + 2$, we have $-du = e^{-x} dx$, and $u = 3$ (resp. $u = 2$) when $x = 0$ (resp. $x = \infty$). Thus

$$\begin{aligned}
\int_{0}^{\infty} \frac{e^{-x}}{(e^{-x}+2)^2} dx &= -\int_{3}^{2} \frac{1}{u^2} du \\
&= -\left[-\frac{1}{u} \right]_{3}^{2} \\
&= \frac{1}{2} - \frac{1}{3} \\
&= \frac{1}{6}.
\end{aligned}$$

□