Problem Set 4 Solutions

MATH 16B Spring 2016

3 March 2015

Exercise (9.2.16)**.** Evaluate

$$\int x^5 \ln(x)\, dx$$

Solution. This would be easier to integrate if we could change $\ln(x)$ to $\frac{1}{x}$ by differentiating it. In fact we can do this using integration by parts. Choose

$$u = \ln x, \quad du = \frac{1}{x}dx, \quad v = \frac{x^6}{6}, \quad dv = x^5 dx.$$

Then integration by parts tells us

$$\begin{aligned}
\int x^5 \ln(x)\, dx &= \frac{x^6}{6}\ln x - \int \frac{x^6}{6}\cdot\frac{1}{x}dx \\
&= \frac{x^6}{6}\ln x - \int \frac{x^5}{6}dx \\
&= \frac{x^6}{6}\ln x - \frac{x^6}{36} + c.
\end{aligned}$$

□

Exercise (9.1.12)**.** Evaluate

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx.$$

Solution. Observe that $\sqrt{x}$ and its (nearly) derivative $\frac{1}{\sqrt{x}}$ both appear, so we use the substitution $u = \sqrt{x}$. Then $2du = \frac{1}{\sqrt{x}}dx$, so

$$\begin{aligned}
\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx &= \int 2e^u du \\
&= 2e^u + c \\
&= 2e^{\sqrt{x}} + c.
\end{aligned}$$

□

Exercise (9.1.25). Evaluate

$$\int \frac{1}{x \ln(x^2)} dx.$$

Solution. The derivative of $\ln(x^2)$ is $\frac{1}{x^2} \cdot 2x = \frac{2}{x}$, so we can use the substitution $u = \ln(x^2)$. Then $\frac{1}{2} du = \frac{1}{x} dx$, so

$$\begin{aligned} \int \frac{1}{x \ln(x^2)} dx &= \int \frac{1}{2} \cdot \frac{1}{u} du \\ &= \frac{1}{2} \ln(u) + c \\ &= \frac{1}{2} \ln(\ln(x^2)) + c. \end{aligned}$$

□

Exercise (9.2.6). Evaluate

$$\int x^2 e^x dx.$$

Solution. We could integrate this if only the x^2 were gone, and we can disappear it by integrating by parts twice. Set

$$u = x^2, \quad du = 2x dx, \quad v = e^x, \quad dv = e^x dx.$$

Then integration by parts tells us

$$\int x^2 e^x dx = x^2 e^x - \int 2x e^x dx.$$

Now set

$$u = 2x, \quad du = 2dx, \quad v = e^x, \quad dv = e^x dx,$$

and integrating by parts again gives

$$\begin{aligned} x^2 e^x - \int 2x e^x dx &= x^2 e^x - 2x e^x + \int 2 e^x dx \\ &= x^2 e^x - 2x e^x + 2e^x + c. \end{aligned}$$

□

Exercise (9.1.42). Evaluate

$$\int (1 + \ln x) \sin(x \ln x) dx.$$

(There is a hint for this exercise in the book).

Solution. The derivative of $x \ln x$ is $x \cdot \frac{1}{x} + \ln x = 1 + \ln x$, so we can use the substitution $u = x \ln x$. Then $du = (1 + \ln x) dx$, so

$$\begin{aligned} \int (1 + \ln x) \sin(x \ln x) dx &= \int \sin u \, dx \\ &= -\cos u + c \\ &= -\cos(x \ln x) + c. \end{aligned}$$

□

Exercise (9.2.39)**.** Evaluate

$$\int \frac{xe^x}{(x+1)^2} dx.$$

(There is a hint for this exercise in the book).

Solution. We'll use integration by parts. Set

$$u = xe^x, \quad du = xe^x + e^x = (x+1)e^x, \quad v = \frac{-1}{x+1}, \quad dv = \frac{1}{(x+1)^2} dx.$$

Then integration by parts tells us

$$\begin{aligned}
\int \frac{xe^x}{(x+1)^2} dx &= -\frac{xe^x}{x+1} - \int (x+1)e^x \cdot \frac{-1}{x+1} dx \\
&= -\frac{xe^x}{x+1} + \int e^x dx \\
&= -\frac{xe^x}{x+1} + e^x + c \\
&\left(= \frac{e^x}{x+1} + c \right).
\end{aligned}$$

□