Problem Set 3 Solutions

MATH 16B Spring 2016

18 February 2015

Exercise (7.6.4)**.** Calculate
$$\int_0^1 \int_{-1}^2 \frac{y^3}{3} dy\, dx.$$

Solution. Using the power rule a couple times, we find
$$\begin{aligned}\int_0^1 \int_{-1}^2 \frac{y^3}{3} dy\, dx &= \int_0^1 \left[\frac{y^4}{12}\right]_{y=-1}^2 dx \\ &= \int_0^1 \frac{5}{4} dx \\ &= \left[\frac{5}{4}x\right]_{x=0}^1 \\ &= \frac{5}{4}.\end{aligned}$$

□

Exercise (7.6.8)**.** Calculate
$$\int_0^1 \int_0^x e^{x+y} dy\, dx$$

Solution. Recall the rule $\int e^{ax+b} dx = \frac{1}{a}e^{ax+b}$. Using this rule,
$$\begin{aligned}\int_0^1 \int_0^x e^{x+y} dy\, dx &= \int_0^1 \left[e^{x+y}\right]_{y=0}^x dx \\ &= \int_0^1 e^{2x} - e^x dx \\ &= \left[\frac{1}{2}e^{2x} - e^x\right]_{x=0}^1 \\ &= \frac{1}{2}e^2 - e + \frac{1}{2}.\end{aligned}$$

□

Exercise (7.6.14)**.** Find the volume of the region bounded above by $f(x, y) = x^2 + y^2$ and lying over the region R bounded by the curves
$$x = 0, \qquad x = 1, \qquad y = 0, \qquad y = \sqrt[3]{x}.$$

Solution. Since one of the boundary curves gives y as a function of x let's integrate y first. From thinking or drawing a picture we see that $y = 0$ is the lower bound and $y = \sqrt[3]{x}$ is the upper bound. Then range of x-values over which we want to integrate is $x = 0$ to $x = 1$. The result is

$$\begin{aligned}\int_0^1 \int_0^{\sqrt[3]{x}} x^2 + y^2 dy\, dx &= \int_0^1 \left[x^2 y + \frac{y^3}{3}\right]_{y=0}^{\sqrt[3]{x}} dx \\ &= \int_0^1 x^{7/3} + \frac{x}{3} dx \\ &= \left[\frac{3}{10}x^{10/3} + \frac{x^2}{6}\right]_{x=0}^{1} \\ &= \frac{7}{15}.\end{aligned}$$

□

Exercise. Let R be the region bounded by the curves

$$y = 1, \qquad y = 4, \qquad y = x^2.$$

Calculate

$$\iint_R x^2 + y\, dx\, dy.$$

Solution. Let's integrate with respect to x first, because there's sort of "three lower bounds" in the y-direction (first $y = x^2$, then $y = 1$, then $y = x^2$ again) and we don't want to deal with that. The left bound for x is $-\sqrt{y}$ and the right bound is $\sqrt{y}$; the y-bounds are then just 1 and 4. So we get

$$\begin{aligned}\int_1^4 \int_{-\sqrt{y}}^{\sqrt{y}} x^2 + y\, dx\, dy &= \int_1^4 \left[\frac{x^3}{3} + xy\right]_{x=-\sqrt{y}}^{\sqrt{y}} dy \\ &= \int_1^4 \frac{8}{3} y^{3/2} dy \\ &= \left[\frac{16}{15} y^{5/2}\right]_{y=1}^{4} \\ &= \frac{16}{15}2^5 - \frac{16}{15} \\ &= \frac{496}{15} \text{ (if you want to simplify it).}\end{aligned}$$

□