Problem Set 1 Solutions

MATH 16B Spring 2016

4 February 2015

Exercise (7.1.5)**.** Let $f(x,y) = xy$. Show that $f(2+h,3) - f(2,3) = 3h$.

Solution. Just by plugging in,

$$f(2+h,3) - f(2,3) = (2+h)(3) - (2)(3) = 6 + 3h - 6 = 3h.$$

□

Exercise (7.2.20)**.** Let $f(x,y) = (x+y^2)^3$. Evaluate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at $(x,y) = (1,2)$.

Solution. Using the chain rule,

$$\frac{\partial f}{\partial x} = 3(x+y^2)^2$$

and

$$\frac{\partial f}{\partial y} = 3(x+y^2)^2 \cdot 2y.$$

Plugging in $(x,y) = (1,2)$, we find

$$\frac{\partial f}{\partial x} = 3(1+2^2)^2 = 75 \qquad \text{and} \qquad \frac{\partial f}{\partial y} = 3(1+2^2)^2 \cdot 2 \cdot 2 = 300.$$

□

Exercise (7.2.24)**.** Let $f(x,y) = xe^y + x^4 y + y^3$. Find $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial y^2}$, $\frac{\partial^2 f}{\partial x \partial y}$, and $\frac{\partial^2 f}{\partial y \partial x}$.

Solution. First we compute

$$\frac{\partial f}{\partial x} = e^y + 4x^3 y$$

and

$$\frac{\partial f}{\partial y} = xe^y + x^4 + 3y^2.$$

The second partial derivatives are just appropriate partial derivatives of these, and we find

$$\frac{\partial^2 f}{\partial x^2} = 12x^2 y,$$
$$\frac{\partial^2 f}{\partial y^2} = xe^y + 6y,$$
$$\frac{\partial^2 f}{\partial x \partial y} = e^y + 4x^3,$$
$$\frac{\partial^2 f}{\partial y \partial x} = e^y + 4x^3.$$

□

Exercise (7.3.7)**.** Find all points (x, y) where $f(x, y) = \frac{1}{3}x^3 - 2y^3 - 5x + 6y - 5$ has a possible relative maximum or minimum.

Solution. By the first derivative test, $f(x, y)$ can only have a maximum or minimum where both $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$. First we compute the first derivatives:

$$\frac{\partial f}{\partial x} = x^2 - 5 \qquad \text{and} \qquad \frac{\partial f}{\partial y} = -6y + 6.$$

Setting these equal to zero we get a system of equations

$$\begin{aligned} x^2 - 5 &= 0 \\ -6y + 6 &= 0. \end{aligned}$$

Solving this system gives two points $(\sqrt{5}, 1)$ and $(-\sqrt{5}, 1)$. These two points are the only possible locations for a maximum or minimum of $f(x, y)$. □

Exercise (7.3.23)**.** Find all points (x, y) where $f(x, y) = x^3 - y^2 - 3x + 4y$ has a possible relative maximum or minimum. Then use the second derivative test at each point to determine, if possible, whether f has a maximum, minimum, or neither.

Solution. The first derivatives are

$$\frac{\partial f}{\partial x} = 3x^2 - 3 \qquad \text{and} \qquad \frac{\partial f}{\partial y} = -2y + 4.$$

Setting them equal to zero gives the system of equations

$$\begin{aligned} 3x^2 - 3 &= 0 \\ -2y + 4 &= 0 \end{aligned}$$

whose solutions are the points $(1, 2)$ and $(-1, 2)$. By the first derivative test, these are the points we need to check for maxima and minima.

The second derivatives are

$$\frac{\partial^2 f}{\partial x^2} = 6x, \qquad \frac{\partial^2 f}{\partial x \partial y} = 0, \qquad \frac{\partial^2 f}{\partial y^2} = -2.$$

So

$$D = \frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 = -12x.$$

At $(x, y) = (1, 2)$ we find $D = -12 < 0$, so by the second derivative test $(1, 2)$ is neither a max nor a min.

At $(x, y) = (-1, 2)$ we find $D = 12 > 0$ and $\frac{\partial^2 f}{\partial x^2} = -6 < 0$, so by the second derivative test $(-1, 2)$ is a max. □