Midterm 1 Review

MATH 16B Spring 2016

Exercise 1. Compute

$$\int_2^5 \int_0^1 9x^2y^2 dy\, dx$$

Exercise 2. Let D be the region of the x, y-plane where x and y are greater than or equal to zero and their sum is at most 3. Find the volume of the solid bounded above by $f(x, y) = e^x$ and lying over D.

Exercise 3. Find the maximum value of

$$h(x, y) = x + 2y - x^2 + xy - y^2.$$

(Note that this asks for the maximum value, not the location where the maximum occurs).

Exercise 4. Compute $\frac{\partial^2 g}{\partial x \partial y}$ and $\frac{\partial^2 g}{\partial y \partial x}$ for

$$g(x, y) = x^y$$

and observe that they are equal.

Exercise 5. Maximize

$$f(x, y, z) = 3x - 3y - 8z - 2x^2 + xy - z^2$$

with respect to the constraint

$$g(x, y, z) = -x + y + 3z = -1.$$

Exercise 6. Compute

$$\int_0^{\pi/4} \int_x^{2x} \cos y\, dy\, dx.$$

Exercise 7. State precisely the first and second derivative tests for functions of two variables.

Exercise 8. Find all maxima and minima of

$$f(x, y) = 2x^2 - x^4 - y^2.$$

Exercise 9. Find all possible points where

$$g(x, y, z) = 3x + 3y - z - x^2 + xy - y^2 - z^2$$

could have a maximum.

Exercise 10. Compute all first and second partial derivatives of

$$f(x, y) = \sin x \sin y + \cos 2xy$$

Exercise 11. The function

$$f(x, y) = 4x + 3y - 1$$

has one maximum and one minimum with respect to the constraint

$$x^2 + y^2 = 25.$$

Find the two points where the maximum and minimum occur.

Exercise 12. Let R be the region bounded by the curves

$$y = x, \qquad x = \sqrt{y}.$$

Compute

$$\iint_R xy \, dy \, dx.$$

Exercise 13. Compute $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and $\frac{\partial f}{\partial z}$ for

$$f(x, y, z) = x^2 y + 3z + xe^{y^2 z}.$$

Exercise 14. If three positive numbers sum to 9, what is the largest their product can be?